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THESIS

Performance of Fast Frequency-Hopped Noncoherent MFSK Conventional and Self-Normalization Receiver over Rician- and Rayleigh-faded channel with Partial-Band Interference

by

Lee, Kang Yeun

Thesis Advisor: R. Clark Robertson
Co-Advisor: Tri T. Ha

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Performance of Fast Frequency-Hopped Noncoherent MFSK Conventional and Self-Normalization Receivers over Rician- and Rayleigh-faded channel with Partial-Band Interference

by

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Submitted in partial fulfillment of the
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ABSTRACT

An error probability analysis is performed for an M-ary orthogonal frequency shift keying (MFSK) receiver employing fast frequency-hopped (FH) spread spectrum waveforms transmitted over a fading channel with partial-band interference. The partial-band interference is modeled as a Gaussian process. Wideband thermal noise is also included in the analysis. Diversity is performed using multiple hops per data bit. Each diversity reception is assumed to fade independently according to a nonselective Rician process. A nonlinear combination procedure referred to as self-normalization combining is employed by the receiver to minimize partial-band interference effects.

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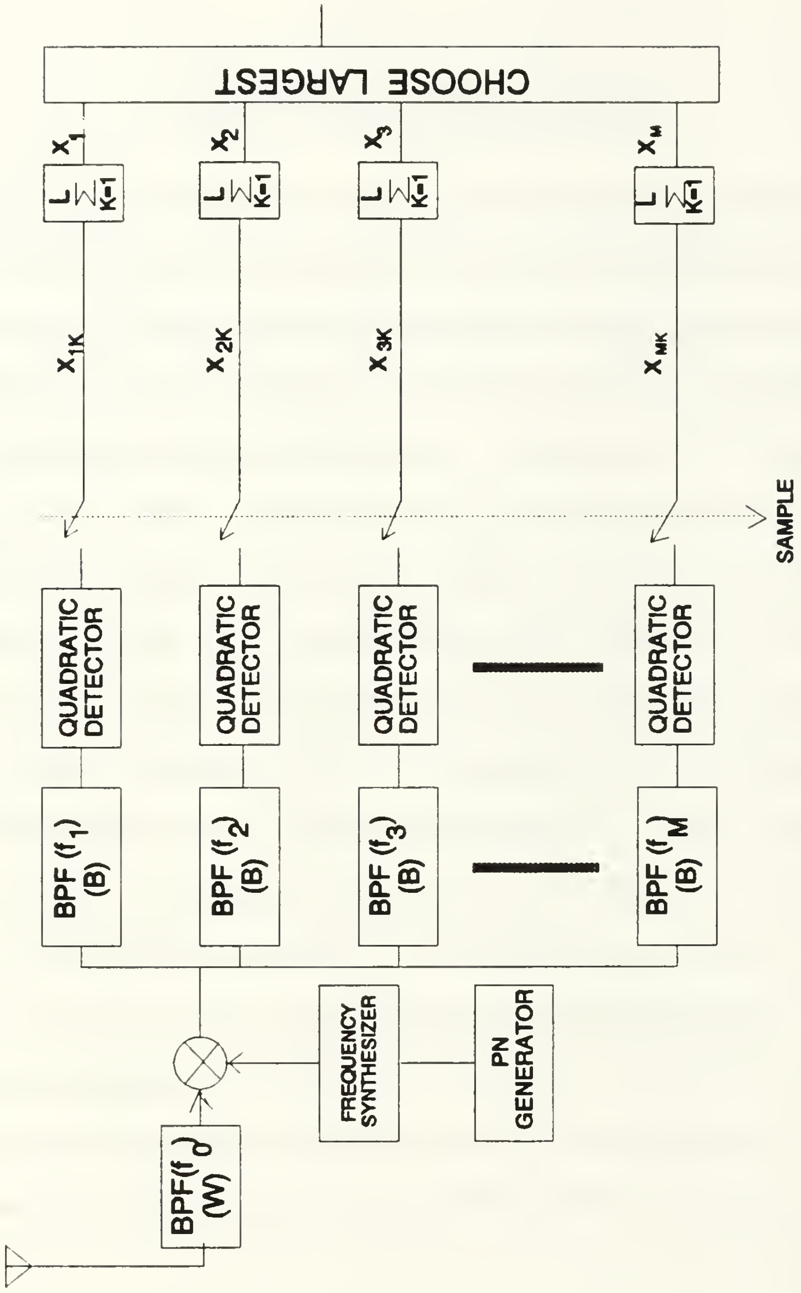
I. INTRODUCTION

Spread spectrum communication systems are designed to use signals having a much wider bandwidth than ordinary communication systems. Some of advantages of spread spectrum signals are the energy gain achivable against a narrow band jammer, low probability of detection (LPD), low probability of intercept (LPI), multiple access operation, and the capability to overcome the effects of multipath [Ref. 1]. The primary disadvantages of spread spectrum signals are the time and frequency synchronization required by the receiver which make it difficult to implement the system and the wide frequency band requirement. There are three primary spread spectrum communication methods: time hopping (TH), frequency hopping (FH) and direct spreading (DS). This thesis presents an error probability analysis of two fast frequency-hopped M-ary orthogonal frequency-shift keyed (FFH/MFSK) systems, where the term 'fast' implies that the frequency-hopping rate is greater than the symbol rate. The first to be examined is a conventional FFH/MFSK system and, the second is a self-normalization FFH/MFSK system. Both systems use noncoherent detection. The channel is assumed to experience both fading and partial-band interference. The performance of the two receivers is compared with each other as well as with another type of FFH/MFSK receiver referred to as a noise-normalization receiver [Ref. 2].

A. Conventional Receiver and Self-Normalization Receiver

In the conventional FFH/MFSK receiver, illustrated in Fig. 1.1, the input signal passes through a wideband radio frequency (RF) filter and is mixed with the frequencies which are created pseudo-randomly in the frequency synthesizer to dehop



Conventional FFH/MFSK Receiver

Figure 1.1: Block Diagram of Conventional FFH/MFSK noncoherent Receiver

and convert the signal frequency down to intermediate frequency (IF). The bandwidth of the RF filter (W) must be wide enough to pass all possible hopping frequencies, while the bandwidth of the IF filter (B) needs pass only one hopping frequency. Thus, the bandwidth of the RF filter (W) is approximately equal to NB, where N is number of the hopping frequencies. The receiver has M different branches for each of the M different symbols. The IF signal is demodulated with a bandpass filter and a quadratic detector. The output signal from each quadratic detector is sampled L times and combined to provide a decision statistic between the M available outputs. The transmitter uses fast frequency-hopping to provide L independent samples through frequency diversity. The receiver is assumed to dehop the signal with perfect timing.

In the self-normalization receiver, illustrated in Fig. 1.2, each of the M quadratic detector outputs are divided by the sum of the quadratic detector outputs from all M channels. This normalization factor is the only difference from the conventional receiver. When partial-band interference present, the hops that are corrupted by the partial-band noise jamming dominate the performance of the conventional detector since each hop is equally weighted in the overall decision statistic. The normalization factor minimizes this effect in the self-normalization receiver. As with the conventional FFH/MFSK receiver, the L samples at the output of the M quadratic detectors are combined to provide a decision statistic between the M available quadratic detector outputs, and the receiver is assumed to dehop the signal with perfect timing.

B. Fading

For both receivers investigated in this thesis, the received signal is assumed to be a combination of a single, nonfaded (direct) component and many reflected (diffuse) signals. By modelling the fading as Rician, in which the signal is considered to consist of a direct signal component and a diffuse signal component, we obtain a

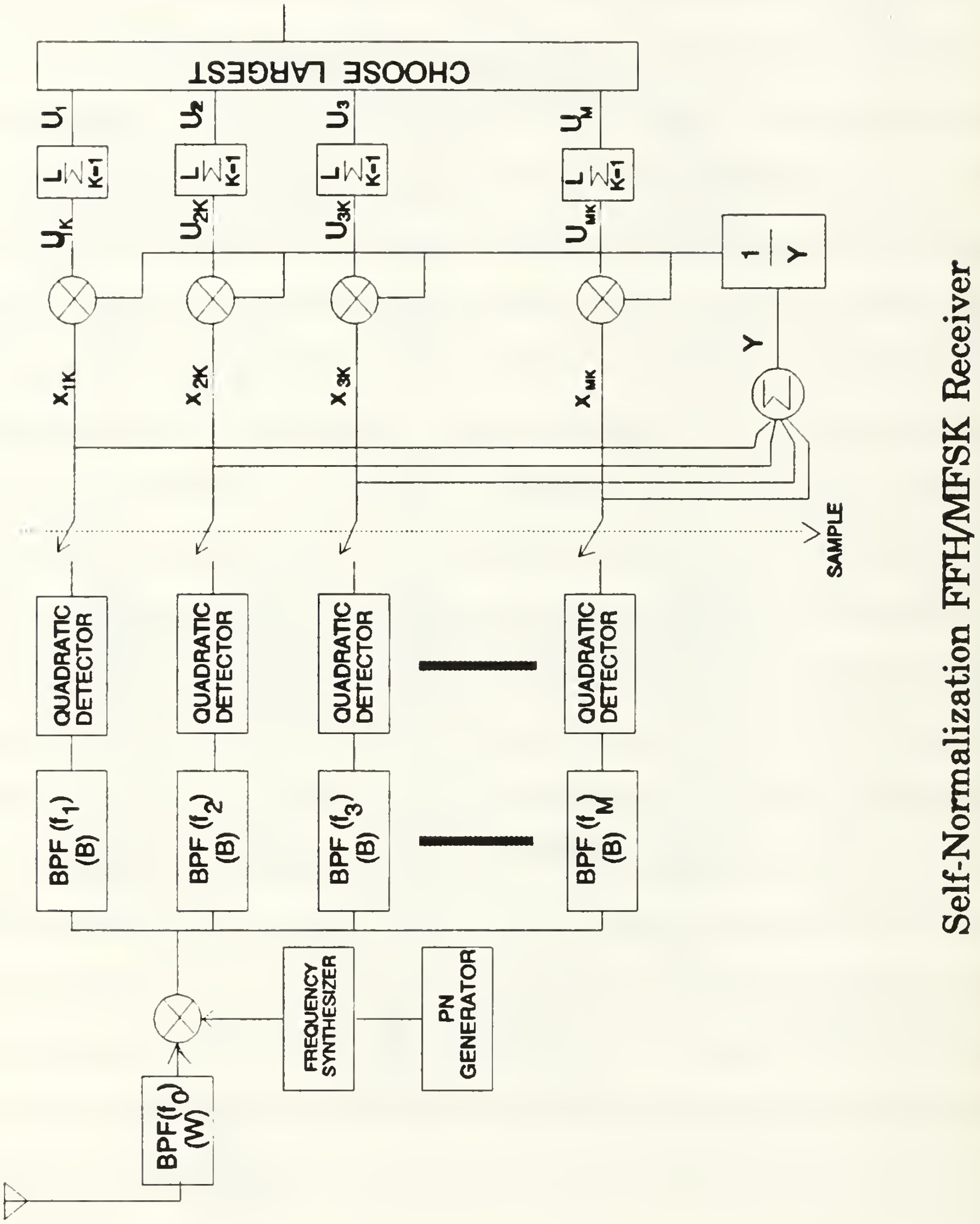


Figure 1.2: Block Diagram of Self-Normalization FFH/MFSK noncoherent Receiver

general result that is valid in the limit of large direct-to-diffuse signal power ratios for Gaussian channels and in the limit of small direct-to-diffuse signal power ratios for Rayleigh fading channels as well as the general case where the effects of both the direct and diffuse components of the signal must be included in the analysis. For a Rician faded signal with amplitude $\sqrt{2}a_k$, the probability density function is [Ref. 3]

$$f_{A_k}(a_k) = \frac{a_k}{2\sigma^2} \exp\left(-\frac{a_k^2 + \alpha_k^2}{2\sigma^2}\right) I_0\left(\frac{a_k \alpha_k}{\sigma^2}\right) u(a_k) \tag{1.1}$$

where α_k^2 is the power of the direct signal component, $2\sigma^2$ is the power of the diffuse signal component, $u(\bullet)$ is the unit step function, and $I_0(\bullet)$ is the zeroth-order modified Bessel function of the first kind. When the direct signal component power is zero, which occurs in times of deep channel fading on the channel such as when the direct communication path is blocked by terrain or other obstacles, the probability density function reverts to the Rayleigh probability density function [Ref. 3]

$$f_{A_k}(a_k) = \frac{a_k}{\sigma^2} \exp\left(-\frac{a_k^2}{2\sigma^2}\right) u(a_k) \tag{1.2}$$

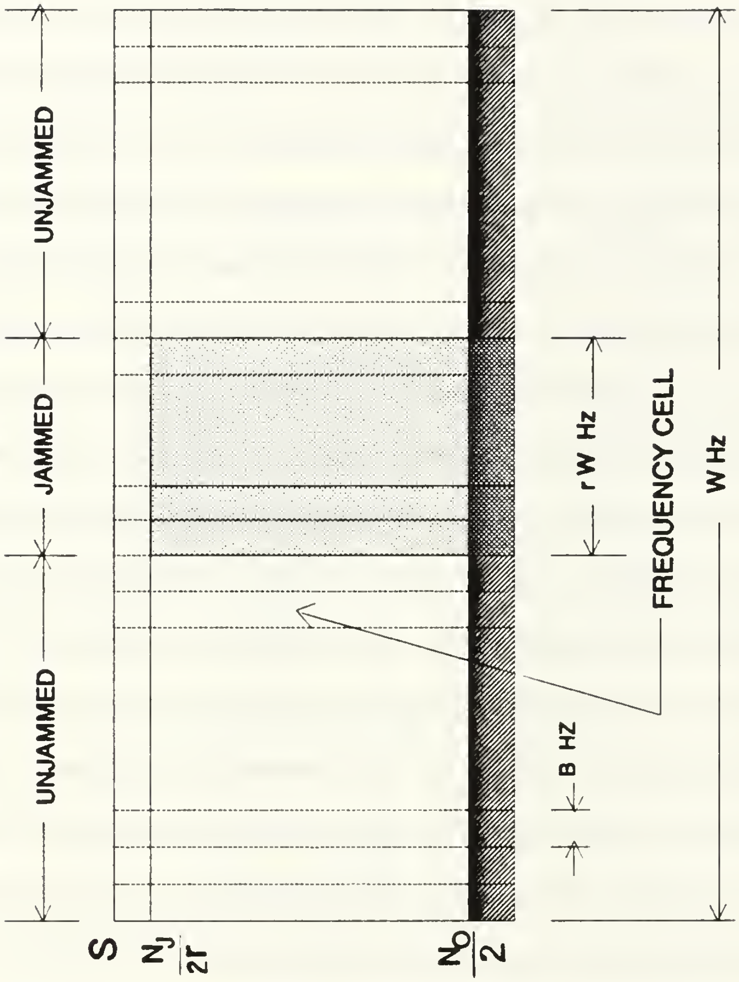
Also in this thesis, we assume that the smallest spacing between frequency hop slots is larger than the coherence bandwidth of the channel [Ref. 4]. As a result, each hop of a symbol experiences independent fading. In addition, the channel for each hop of a symbol is modeled as a frequency-nonselective, slowly fading Rician process. This implies that the signal bandwidth is much smaller than the coherence bandwidth of the channel and that the hop duration is much smaller than the coherence time of the channel [Ref. 5]. The latter assumption is equivalent to requiring the hop rate to be large compared to the Doppler spread of the channel. As a result, the signal amplitude can be modeled as a Rician random variable that remains fixed at least for the duration of a single hop.

C. Partial-Band Interference

One of the applications of frequency-hopped spread spectrum signals is to reduce receiver performance degradation due to narrowband interference (either intentional or otherwise). This is accomplished by transmitting each symbol at a different carrier frequency according to some apparently random pattern known only by the transmitter and the receiver. The bandwidth of the frequency-hopped signal is generally much larger than the signal bandwidth in the absence of frequency-hopping. Consequently, in order to interfere with the entire spread spectrum band, given a fixed total interference power the noise power spectral density of the narrowband interference is reduced. Rather than interfere with the entire spread-spectrum bandwidth simultaneously with the consequent reduction in noise power spectral density, the narrowband interference may affect only a portion of the total spread spectrum bandwidth at any one time, and the portion of the total spread spectrum bandwidth experiencing narrowband interference may also change in an apparently random manner. This is referred to as partial-band interference. In the case of partial-band interference, only some of the transmitted symbols will be affected by narrowband interference, and the question arises as to whether performance can be improved by implementing diversity in the form of fast frequency-hopping; that is, each information symbol is transmitted multiple times, and each transmission is at a different apparently random carrier frequency within the overall spread spectrum bandwidth.

As mentioned previously, in this thesis the performance of two different fast frequency-hopped M-ary orthogonal frequency-shift keyed (FFH/MFSK) noncoherent receivers is investigated. The performance of the conventional FFH/MFSK noncoherent receiver in a Rician fading channel with only wideband noise (no partial-band interference) has been previously investigated [Ref. 6]. For the self-normalized FFH/MFSK receiver, system performance is obtained as an upper bound on the probability of bit error, while for the conventional FFH/MFSK receiver, the actual probability of bit error is obtained. For both systems, the transmitter is assumed to implement

In addition to wideband Gaussian noise, the channel is assumed to be affected by narrowband noise in the form of partial-band interference. The interference that is considered in this thesis may be due to a partial-band jammer as well as other unintended narrowband interferences. The partial-band interference is modeled as additive Gaussian noise and is assumed to corrupt only a fraction γ, where $1 \geq \gamma \geq 0$, of the entire spread spectrum bandwidth at any one time. This is illustrated in Fig. 1.3 where S represents the signal power level, $N_j/2$ the jamming noise plus thermal noise power spectral density, $N_0/2$ the thermal noise power spectral density, B the bandwidth of one frequency cell, and W the entire frequency band. In addition, partial-band interference, when present, is assumed to be present in each branch of the MFSK demodulator, and the fraction of the spread spectrum bandwidth experiencing partial-band interference is assumed to be the same for all hops of a symbol. Hence, γ is the probability that partial-band interference is present in all M channels of the receiver, and $1 - \gamma$ is the probability that partial-band interference is not present in all M channels of the receiver. If $N_j/2$ is defined to be the average power spectral density of narrowband interference over the entire spread spectrum bandwidth, then $\gamma^{-1}N_j/2$ is the power spectral density of partial-band interference when it is present. Thermal noise and other wideband noise that corrupt the channel are modeled as additive white Gaussian noise, and the power spectral density of this wideband noise is defined as $N_0/2$. Thus, the power spectral density of the total noise is $\gamma^{-1}N_j/2+N_0/2$ when partial-band interference is present and $N_0/2$ otherwise. If the equivalent noise bandwidth of each bandpass filter in both the conventional and the self-normalized FFH/MFSK demodulators is B Hz, then for each hop the signal is received with noise



Thermal Noise and Partial-Band Jamming Noise Model ($r<1$)

Figure 1.3: Thermal Noise and Partial-Band Jamming Noise Model

of power N_0B with probability $1-\gamma$ when partial-band interference is not present and with noise of power $(\gamma^{-1}N_j + N_0)B$ with probability γ when partial-band interference is present.

The symbol rate is $R_s = R_b/\log_2 M$ where M is the order of the MFSK modulation and R_b is the bit rate. Since the MFSK signal is assumed to perform L hops per symbol, the hop rate is $R_h = LR_s$. The equivalent noise bandwidth of the bandpass filters in each of the M channels of both the self-normalized FFH/MFSK receiver and the conventional FFH/MFSK receiver must be at least as wide as the hop rate, and in this thesis we use $B = R_h$. The overall system bandwidth is assumed to be very large compared to the hop rate.

D. Fast Frequency-Hopping

There are two types of frequency-hopping: fast frequency-hopping and slow-frequency hopping. In fast-frequency hopping the carrier frequency changes (hops) more than once per data symbol and all hops within a symbol duration are combined in the receiver to provide the sample statistic, but in slow frequency-hopping the carrier frequency changes only once per data symbol. Hence, the terms of 'slow' and 'fast' do not mean absolutely how fast the carrier frequency changes. We define diversity of order L as the case where each M-ary symbol is transmitted L times over the channel. Thus, if E_h is the energy for each trasmission then each M-ary symbol requires a total energy of

$$E_s = LE_h \tag{1.3}$$

where E_s is the symbol energy. The diversity factor is a key point of fast frequency-hopping communication systems, and the object of this thesis is to determine under what conditons fast frequency-hopping systems can provide immunity to partial-band interference.

II. ANALYSIS

A. Determination of the Probability of Bit Error

1. Conventional Receiver

An analysis of the conventional FFH/MFSK receiver requires, to begin with, a description of the samples at the output of the M quadratic detectors before diversity combining. The partial-band interference, when present, is assumed to be present in all channels. Assuming equally likely M-ary symbols, we have the probability of symbol error given L independent hops as

$$P_s = \sum_{i=0}^{L} \binom{L}{i} \gamma^i (1-\gamma)^{L-i} P_s(i) \tag{2.1}$$

where $P_s(i)$ is the conditional symbol error probability given that i out of L hops are jammed. Assuming the signal is present in branch one, we have the probability density function of the random variable x_{1k} representing the output of the quadratic detector [Ref. 3].

$$f_{X_{1k}}(x_{1k}|a_k) = \frac{1}{2\sigma_k^2} \exp\left(-\frac{x_{1k}+2a_k^2}{2\sigma_k^2}\right) I_0\left(\frac{a_k\sqrt{2x_{1k}}}{\sigma_k^2}\right) u(x_{1k}) \tag{2.2}$$

where $\sqrt{2}a_k$ is the signal amplitude, σ_k^2 is noise power in hop k, and $I_0(\bullet)$ is the zeroth-order modified Bessel function of the first kind. The unconditional probability density function is found by evaluating

$$f_{X_{1k}}(x_{1k}) = \int_0^\infty f_{X_{1k}}(x_{1k}|a_k) f_{A_k}(a_k)\, da_k \tag{2.3}$$

The amplitude of the signal is modeled as a Rician distributed random variable with a probability density function as given by (1.1). Substituting (1.1) and (2.2) into (2.3), we have

$$f_{X_{1k}}(x_{1k}) = \frac{1}{2(\sigma_k^2+2\sigma^2)} \exp\left(-\frac{x_{1k}+2a_k^2}{2(\sigma_k^2+2\sigma^2)}\right) I_0\left(\frac{a_k\sqrt{2x_{1k}}}{\sigma_k^2+2\sigma^2}\right) u(x_{1k}) \tag{2.4}$$

The probability density function of each of the $M-1$ channels with only Gaussian noise is

$$f_{X_{mk}}(x_{mk}) = \frac{1}{2\sigma_k^2} \exp\left(-\frac{x_{mk}}{2\sigma_k^2}\right) u(x_{1k}) \tag{2.5}$$

which is easily obtained by replacing the signal amplitude a_k with 0 and replacing x_{1k} with x_{mk}, where $m = 2, 3, 4 \ldots M$. Next, the L samples are combined to obtain the final sample statistic. Assuming i out of L hops are jammed, we obtain the decision variable for branch 1

$$\begin{aligned} X_1 &= \sum_{k=1}^{L} X_{1k} \\ &= \sum_{k=1}^{i} X_{1k}^{(1)} + \sum_{k=i+1}^{L} X_{1k}^{(2)} \end{aligned} \tag{2.6}$$

where the superscript $^{(1)}$ implies a jammed hop and $^{(2)}$ implies a non-jammed hop. Since each hop is assumed to be independent, we have

$$f_{X_1}(x_1) = \left[f_{X_{1k}^{(1)}}(x_{1k}^{(1)})\right]^{\otimes i} \bigotimes \left[f_{X_{1k}^{(2)}}(x_{1k}^{(2)})\right]^{\otimes (L-i)} \tag{2.7}$$

where $f_{X_{1k}^{(1)}}(x_{1k}^{(1)})$ is the probability density function when jamming is present, $f_{X_{2k}^{(2)}}(x_{2k}^{(2)})$ when no jamming is present, $\otimes$ implies convolution, $\otimes i$ implies an $i-$fold convolution, and $\otimes(L-i)$ implies an $(L-i)$-fold convolution. Similarly, for the noise only branches, we have the random variables

$$\begin{aligned} X_m &= \sum_{k=1}^{L} X_{mk} \\ &= \sum_{k=1}^{i} X_{mk}^{(1)} + \sum_{k=i+1}^{L} X_{mk}^{(2)} \qquad m = 2, 3, \cdots, M \end{aligned} \tag{2.8}$$

and the probability density function

$$f_{X_m}(x_m) = \left[f_{X_{mk}^{(1)}}(x_{mk}^{(1)})\right]^{\otimes i} \otimes \left[f_{X_{mk}^{(2)}}(x_{mk}^{(2)})\right]^{\otimes (L-i)} \tag{2.9}$$

For MFSK, the probability of symbol error is [Ref. 5]

$$P_s(i) = 1 - \int_0^1 f_{X_1}(x_1|i) \left[\int_0^{x_1} f_{X_m}(x_m) dx_m\right]^{M-1} dx_1 \tag{2.10}$$

Equation (2.10) requires $f_{X_1}(x_1|i)$ and $f_{X_m}(x_m)$. In order to obtain $f_{X_1}(x_1|i)$, we first rewrite (2.7)

$$f_{X_1}(x_1|i) = f^1_{X_1}(x_1|i) \otimes f^2_{X_1}(x_1|i) \tag{2.11}$$

Two probability density functions are computed separately

$$\begin{aligned} f^1_{X_1}(x_1|i) &= \left[f^{(1)}_{X_{1k}}(x^{(1)}_{1k}|i)\right]^{\otimes i} \\ &= \mathcal{L}^{-1}\left\{\left[F^1_{X_{1k}}(s)\right]^i\right\} \end{aligned} \tag{2.12}$$

where $F_{X_{1k}}(s)$ is Laplace transform of $f_{X_{1k}}(x_{1k}|i)$ and $\mathcal{L}^{-1}$ is the inverse Laplace transform operation.

$$\begin{aligned} \left[F^1_{X_{1k}}(s)\right]^i &= \exp\left[\frac{-2i\alpha_k^2\sigma_{kj}^2(\sigma_{kj}^2+2\sigma^2)}{(1+2s(\sigma_{kj}^2+2\sigma^2))(\sigma_{kj}^2+2\sigma^2)}\right] \\ &\times \left[\frac{1}{1+2s(\sigma_{kj}^2+2\sigma^2)}\right]^i \end{aligned} \tag{2.13}$$

where σ_{kj}^2 represents jamming noise power plus thermal noise power. The inverse Laplace transform of (2.13) yields

$$\begin{aligned} f^1_{X_1}(x_1|i) &= \exp\left[-\frac{2i\alpha_k^2+x_1}{2(\sigma_{kj}^2+2\sigma^2)}\right]\left[\frac{x_1(\sigma_k^2+2\sigma^2)}{i\alpha_k^2}\right]^{\frac{i-1}{2}} \\ &\times \left[\frac{1}{2(\sigma_{kj}^2+2\sigma^2)}\right]^{\frac{i+1}{2}} I_{i-1}\left[\frac{\sqrt{2i\alpha_k^2x_1}}{\sigma_{kj}^2+2\sigma^2}\right] \end{aligned} \tag{2.14}$$

Similarly

$$\begin{aligned} f^2_{X_1}(x_1|i) &= \exp\left[-\frac{2(L-i)\alpha_k^2+x_1}{2(\sigma_k^2+2\sigma^2)}\right]\left[\frac{x_1(\sigma_k^2+2\sigma^2)}{(L-i)\alpha_k^2}\right]^{\frac{L-i-1}{2}} \\ &\times \left[\frac{1}{2(\sigma_k^2+2\sigma^2)}\right]^{\frac{L-i+1}{2}} I_{L-i-1}\left[\frac{\sqrt{2(L-i)\alpha_k^2x_1}}{\sigma_k^2+2\sigma^2}\right] \end{aligned} \tag{2.15}$$

where σ_k^2 represents thermal noise power. In order to obtain $f_{X_1}(x_1|i)$, (2.14) and (2.15) must be convolved. In general, this cannot be done analytically.

For the noise branch, the probability density function is

$$f_{X_m}(x_m) = \left[\frac{1}{2\sigma_{kj}^2} e^{-\frac{x_m}{2\sigma_{kj}^2}}\right]^{\otimes i} \bigotimes \left[\frac{1}{2\sigma_k^2} e^{-\frac{x_m}{2\sigma_k^2}}\right]^{\otimes (L-i)} \tag{2.16}$$

The Laplace transform of (2.16) is

$$F_{X_m}(s) = \left[\frac{\beta_k^{(1)}}{s+\beta_k^{(1)}}\right]^i \left[\frac{\beta_k^{(2)}}{s+\beta_k^{(2)}}\right]^{L-i} \tag{2.17}$$

where

$$\beta_k^{(1)} = \frac{1}{2\sigma_{kj}^2}\ , \quad \beta_k^{(2)} = \frac{1}{2\sigma_k^2} \tag{2.18}$$

Equation (2.17) can be expanded in a Heaviside partial fraction expansion to obtain

$$\begin{aligned} F_{X_m}(s) &= \frac{K_{11}}{s+\beta_k^{(1)}} + \frac{K_{12}}{\left(s+\beta_k^{(1)}\right)^2} + \cdots + \frac{K_{1i}}{\left(s+\beta_k^{(1)}\right)^i} \\ &+ \frac{K_{21}}{s+\beta_k^{(2)}} + \frac{K_{22}}{\left(s+\beta_k^{(2)}\right)^2} + \cdots + \frac{K_{2(L-i)}}{\left(s+\beta_k^{(2)}\right)^{(L-i)}} \end{aligned} \tag{2.19}$$

where the coefficients K_{1j}, $j = 1, 2, \cdots, i$ and K_{1k} , $k = 1, 2, \cdots, L-i$ depend on L and i. Specific coefficients for several values of L and i are given in Table 2.1. Applying the inverse Laplace transform to (2.19), we obtain

$$\begin{aligned} f_{X_m}(x_m) &= e^{-\beta_K^{(1)} x_m}\left(K_{11} + K_{12}x_m + \frac{K_{13}x_m^2}{2!} + \cdots + K_{1i}\frac{x_m^{i-1}}{(i-1)!}\right) \\ &+ e^{-\beta_k^{(2)} x_m}\left(K_{21} + K_{22}x_m + \frac{K_{23}x_m^2}{2!} + \right. \\ &\left. \cdots + K_{2(L-i)}\frac{x_m^{L-i-1}}{(L-i-1)!}\right) \end{aligned} \tag{2.20}$$

We can now compute the second integral in (2.10) using (2.20) with the result

$L,\ i$	*Coefficients*
$L=1,\quad i=0$	$K_{21}=\beta_k^{(2)}$
$L=1,\quad i=1$	$K_{11}=\beta_k^{(1)}$
$L=2,\quad i=0$	$K_{21}=0$ $K_{22}=(\beta_k^{(2)})^2$
$L=2,\quad i=1$	$K_{11}=\frac{\beta_k^{(1)}\beta_k^{(2)}}{-\beta_k^{(1)}+\beta_k^{(2)}}$ $K_{21}=-K_{11}$
$L=2,\quad i=2$	$K_{11}=0$ $K_{12}=(\beta_k^{(1)})^2$
$L=3,\quad i=0$	$K_{21}=0$ $K_{22}=0$ $K_{23}=(\beta_k^{(2)})^3$
$L=3,\quad i=1$	$K_{11}=\beta_k^{(1)}\left(\frac{\beta_k^{(2)}}{-\beta_k^{(1)}+\beta_k^{(2)}}\right)^2$ $K_{22}=\frac{\beta_k^{(1)}(\beta_k^{(2)})^2}{\beta_k^{(1)}-\beta_k^{(2)}}$ $K_{21}=\beta_k^{(1)}\beta_k^{(2)}-K_{11}\beta_k^{(2)}-\frac{K_{22}}{\beta_k^{(2)}}$
$L=3,\quad i=2$	$K_{21}=\left(\frac{\beta_k^{(1)}}{-\beta_k^{(1)}+\beta_k^{(2)}}\right)^2\beta_k^{(2)}$ $K_{12}=\frac{(\beta_k^{(1)})^2\beta_k^{(2)}}{-\beta_k^{(1)}+\beta_k^{(2)}}$ $K_{11}=\beta_k^{(2)}\beta_k^{(1)}-K_{21}\beta_k^{(1)}-\frac{K_{12}}{\beta_k^{(2)}}$
$L=3,\quad i=3$	$K_{11}=0$ $K_{12}=0$ $K_{13}=(\beta_k^{(1)})^3$

TABLE 2.1: Coefficients of Heaviside Partial Fraction Expansion

$$\begin{aligned}\int_0^{x_1} f_{X_m}(x_m)dx_m &= \sum_{p=1}^{i}\frac{K_{1p}}{(\beta_k^{(1)})^p} \\ &- \sum_{p=1}^{i}K_{1p}e^{-\beta_k^{(1)}x_1}\sum_{r=0}^{p=1}\frac{x_1^{i-1-r}}{(i-1-r)!(\beta_k(1))^{r+1}} \\ &+ \sum_{p=1}^{L-i}\frac{K_{2p}}{(\beta_k^{(2)})^p} \\ &- \sum_{p=1}^{L-i}K_{2p}e^{-\beta_k^{(2)}x_1}\sum_{r=0}^{p=1}\frac{x_1^{L-i-1-r}}{(L-i-1-r)!(\beta_k^{(2)})^{r+1}} \qquad (2.21)\end{aligned}$$

So the final form of $P_s(i)$ which must be evaluated numerically is

$$\begin{aligned}P_s(i) &= 1-\int_0^{\infty}\Bigg[\Bigg\{\exp\left(-\frac{2i\alpha_k^2+x_1}{2(\sigma_{kj}^2+2\sigma^2)}\right) \\ &\times\left(\frac{x_1(\sigma_{kj}^2+2\sigma^2)}{i\alpha_k^2}\right)^{\frac{i-1}{2}}\left(\frac{1}{2(\sigma_{kj}^2+2\sigma^2)}\right)^{\frac{i+1}{2}} I_{i-1}\left(\frac{\sqrt{2i\alpha_k^2x_1}}{\sigma_{kj}^2+2\sigma^2}\right)\Bigg\} \\ &\bigotimes\Bigg\{\exp\left(-\frac{2(L-i)\alpha_k^2+x_1}{2(\sigma_k^2+2\sigma^2)}\right)\left(\frac{x_1(\sigma_k^2+2\sigma^2)}{(L-i)\alpha_k^2}\right)^{\frac{L-i-1}{2}} \\ &\times\left(\frac{1}{2(\sigma_k^2+2\sigma^2)}\right)^{\frac{L-i+1}{2}} I_{L-i-1}\left(\frac{\sqrt{2(L-i)\alpha_k^2x_1}}{\sigma_k^2+2\sigma^2}\right)\Bigg\}\Bigg] \\ &\times\Bigg[\sum_{p=1}^{i}\frac{K_{1p}}{(\beta_k^{(1)})^p}-\sum_{p=1}^{i}K_{1p}e^{-\beta_k^{(1)}x_1}\sum_{r=0}^{p=1}\frac{x_1^{i-1-r}}{(i-1-r)!(\beta_k^{(1)})^{r+1}} \\ &+\sum_{p=1}^{L-i}\frac{K_{2p}}{(\beta_k^{(2)})^p}-\sum_{p=1}^{L-i}K_{2p}e^{-\beta_k^{(2)}x_1} \\ &\times\sum_{r=0}^{p=1}\frac{x_1^{L-i-1-r}}{(L-i-1-r)!(\beta_k^{(2)})^{r+1}}\Bigg]^{M-1} dx_1 \qquad (2.22)\end{aligned}$$

2. Self-normalization Receiver

The difference between the self-normalization receiver and the conventional receiver is the self-normalization factor which is the sum of all M channel outputs after sampling at the outlet of the quadratic detectors. The new decision statistic is the combination of self-normalized quadratic detector outputs. In order to find the probability density function of the random variables representing the self-normalized quadratic detector outputs, we first define the random variable

$$V_k = (X_{2k} + X_{3k} + \cdots + X_{Mk}) - X_{mk} \, , \qquad (m \neq 1) \tag{2.23}$$

The probability density function of the sum of independent random variables is the convolution of the probability density functions of those random variables. Thus, the probability density function of the random variable V_k requires $M-2$ convolutions:

$$\begin{aligned} f_{V_k}(v_k) &= f_{X_{2k}}(x_{2k}) \otimes f_{X_{3k}}(x_{3k}) \otimes \cdots \otimes f_{X_{ik}}(x_{ik}) \otimes \\ & \quad \cdots \otimes f_{X_{Mk}}(x_{Mk}) \quad (i \neq m) \end{aligned} \tag{2.24}$$

Assuming that all the probability density functions of the branches containing only noise are the same, we have

$$f_{V_k}(v_k) = [f_{X_{ik}}(x_{ik})]^{\otimes(M-2)} \qquad (i \neq 1 \quad or \quad m) \tag{2.25}$$

The Laplace transform of $f_{X_{ik}}(x_{ik})$ is

$$F_{X_{ik}}(s) = \frac{1}{2\sigma_k^2 s + 1} \tag{2.26}$$

Hence

$$F_{V_k}(s) = \left(\frac{1}{2\sigma_k^2 s + 1} \right)^{M-2} \tag{2.27}$$

Taking the inverse Laplace transform of (2.27), we get

$$f_{V_k}(v_k) = \left(\frac{1}{2\sigma_k^2} \right)^{M-2} \frac{(v_k)^{M-3}}{(M-3)!} \exp\left(-\frac{v_k}{2\sigma_k^2} \right) \tag{2.28}$$

As can be seen from Fig. 1.2, the random variables that represent the self-normalized outputs of each of the M channels after diversity combining are obtained as

$$U_i = \sum_{k=1}^{L} U_{ik} \, , i = 1, 2, \ldots, M \tag{2.29}$$

where U_{ik}, $i = 1, 2, \ldots, M$, $k = 1, 2, \ldots, L$ are the random variables that represent the self-normalized outputs of each of the M channels for hop k of a symbol. These random variables are expressed in terms of the independent random variables $X_{ik}, i = 1, 2, \ldots, M$, $k = 1, 2, \ldots, L$ representing the M independent outputs of the M quadratic detectors for each channel of the receiver for hop k of a symbol as

$$U_{ik} = \frac{X_{ik}}{X_{1k} + X_{2k} + \ldots + X_{Mk}} \tag{2.30}$$

where $0 \leq U_{ik} \leq 1$ since $0 \leq X_{ik} < \infty, i = 1, 2, \ldots, M$. Using 2.29, we can express $\Pr(U_1 < U_m|i)$ as

$$\Pr(U_1 < U_m|i) = \Pr\left(\sum_{k=1}^{L} U_{1k} < \sum_{k=1}^{L} U_{mk}|i\right) = \Pr\left(\sum_{k=1}^{L} U_{1k} - U_{mk} < 0|i\right) \tag{2.31}$$

Defining the random variable $Z_{1k} = U_{1k} - U_{mk}$, we obtain

$$\begin{aligned} \Pr(U_1 < U_m|i) &= \Pr\left(\sum_{k=1}^{L} Z_{1k} < 0|i\right) \\ &= \Pr(Z_1 < 0|i) \end{aligned} \tag{2.32}$$

where

$$Z_1 = \sum_{k=1}^{L} Z_{1k} \tag{2.33}$$

is our alternative decision variable.

From 2.30 and the definition of Z_{1k} above, we have

$$\begin{aligned} Z_k &= U_{1k} - U_{mk} \\ &= \frac{X_{1k} - X_{mk}}{X_{1k} + X_{2k} + \cdots + X_{Mk}} \\ &= \frac{X_{1k} - X_{mk}}{X_{1k} + X_{mk} + V_k} \end{aligned} \tag{2.34}$$

where $-1 \leq Z_k \leq 1$. This random variable represents the difference between the power in the branch containing the signal and the power of one noise only branch which are detected by quadratic detectors and normalized by the self-normalization factor. We use the auxiliary variable method [Ref. 7] to obtain the probability density function of the new random variable Z_{1k}, where we define the two auxiliary variables

$$W_k = V_k, \qquad Y_k = X_{1k} \tag{2.35}$$

Reforming (2.34) by replacing V_k and X_{1k} with (2.35), we have

$$X_{mk} = \frac{Y_k(1 - Z_{1k}) - Z_{1k}W_k}{1 + Z_{1k}} \tag{2.36}$$

and the Jacobian of the transformation is

$$J = \frac{(1 + z_{1k})^2}{2y_k + w_k} \tag{2.37}$$

Now the joint probability density function for the random variables Z_{1k}, W_k, and Y_k is obtained in terms of the joint probability density function of the independent random variables X_{1k}, X_{mk}, and V_k as

$$\begin{aligned} f_{Z_{1k}Y_kW_k}(z_{1k}, y_k, w_k|i) &= \frac{1}{|J|} f_{X_{1k}}(y_k|i) f_{x_{mk}}\left(\frac{y_k(1 - z_{1k}) - z_{1k}w_k}{1 + z_{1k}}\right) f_{V_k}(w_k) \\ &= \frac{2y_k + w_k}{(1 + z_{1k})^2} \\ &\quad \times \frac{1}{2(\sigma_k^2 + 2\sigma^2)} \exp\left(-\frac{y_k + 2a_k^2}{2(\sigma_k^2 + 2\sigma^2)}\right) I_0\left(\frac{a_k\sqrt{2y_k}}{\sigma_k^2 + 2\sigma^2}\right) \\ &\quad \times \frac{1}{2\sigma_k^2} \exp\left(-\frac{y_k(1 - z_{1k}) + z_{1k}w_k}{2\sigma_k^2(1 + z_{1k})}\right) \\ &\quad \times \left(\frac{1}{2\sigma_k^2}\right)^{M-2} \frac{(w_k)^{M-3}}{(M - 3)!} \exp\left(-\frac{w_k}{2\sigma_k^2}\right) \end{aligned} \tag{2.38}$$

Before evaluating the probability density function of the random variable z_{1k}, we must be careful in handling the range of the variables. Note that the range of the variable X_{mk} is $-\infty < X_{mk} < \infty$, but recall that $f_{X_{mk}}(x_{mk})$ is nonzero only for the range $X_{mk} \geq 0$. For the range of $-1 \leq Z_{1k} \leq 0$, X_{mk} is always greater than zero; but for the range of $0 \leq Z_{1k} \leq 1$, the range of X_{mk} is $-\infty < X_{mk} < \infty$. Consequently, special care must be taken in evaluating $f_{Z_{1k}}(z_{1k})$. The probability density function of the random variable Z_{1k} is obtained from

$$f_{Z_{1k}}(z_{1k}|i) = \int_0^\infty \int_0^\infty \frac{1}{|J|} f_{Z_{1k}Y_kW_k}(z_{1k}, y_k, w_k|i)\,dw_k\,dy_k \tag{2.39}$$

when $-1 \leq Z_{1k} \leq 0$. Since $f_{X_{mk}}(\bullet)$ is nonzero only for positive values of its argument, then

$$f_{Z_{1k}}(z_{1k}) = \int_0^\infty \int_0^{y_k(1-z_{1k})/z_{1k}} \frac{1}{|J|} f_{Z_{1k}Y_kW_k}(z_{1k}, y_k, w_k)\,dw_k\,dy_k \tag{2.40}$$

when $0 \leq Z_{1k} \leq 1$. Evaluating the integrals in (2.39) and (2.40), we have the probability density function of random variable Z_{1k} conditional on i

$$\begin{aligned} f_{Z_{1k}}(z_{1k}|i) \;=\; & \left(\frac{(1+z_{1k})^{M-2}}{2+\xi_k(1-z_{1k})}\right) \exp\left(\frac{-\rho_k(1-z_{1k})}{2+\xi_k(1-z_{1k})}\right) \\ & \left\{ M-2+\frac{2(1+\xi_k)}{2+\xi_k(1-z_{1k})}\left[1+\frac{\rho_k(1+z_{1k})}{(1+\xi_k(1-z_{1k}))(1+\xi_k)}\right]\right\} \\ & -g(z_{1k})u(z_{1k}) \end{aligned} \tag{2.41}$$

where

$$\begin{aligned} g(z_{1k}) \;=\; & \frac{z_{1k}(1-z_{1k})^{M-2}(1+\xi_k)^{M-2}}{(1+\xi_k(1-z_{1k}))^{M-1}(1+z_{1k})} \exp\left(\frac{-\rho_k(1-z_{1k})}{1+\xi_k(1-z_{1k})}\right) \\ & \times \left\{ \frac{1}{(M-3)!}\left(\frac{\rho_k z_{1k}}{(1-\xi_k(1-z_{1k}))(1+\xi_k)}\right)^{M-2} \right. \\ & + \sum_{m=0}^{M-3} \left\{ \frac{(M-2)(M-2)!}{(M-2-m)!(m!)^2} \right. \\ & \times \left(\frac{\rho_k z_k}{(1+\xi_k(1-z_{1k}))(1+\xi_k)}\right)^m + \left(\frac{1+z_{1k}}{1+\xi_k(1-z_{1k})}\right)^m \\ & \times \left\{ \frac{2z_{1k}}{(1-z_{1k})(M-3-m)!}\left[\frac{\rho_k z_{1k}}{(1+\xi_k(1-z_{1k}))(1+\xi_k)}\right]^{M-2-m} \right. \\ & + \sum_{q=0}^{M-3-m} \frac{M-3-m}{(M-3-m-q)!(q!)^2}\left(\frac{\rho_k z_{1k}}{(1+\xi_k(1-z_{1k}))(1+\xi_k)}\right)^q \end{aligned}$$

$$\times \left[\frac{2z_{1k}(M-2-m)^2}{(1-z_{1k})(M-2-m-q)} + \frac{(1+\xi_k(1-z_{1k}))(1+z_{1k})(M-2)}{(1-z_{1k})(1+\xi_k)} \right] \Bigg\} \Bigg\} \Bigg\} \tag{2.42}$$

and

$$\begin{aligned} \rho_k &= \frac{\alpha_k^2}{\sigma_k^2} \\ \xi_k &= \frac{2\sigma^2}{\sigma_k^2} \end{aligned} \tag{2.43}$$

are the signal-to-noise ratios of the direct signal component and the diffuse (or faded) signal component, respectively.

Having obtained the probability density function of the random variable Z_{1k}, we need to consider diversity to get the overall output sample statistics. The random variable Z_1 is defined

$$\begin{aligned} Z_1 &= \sum_{k=1}^{L} Z_{1k} \\ &= \sum_{k=1}^{i} Z_{1k}^{(1)} + \sum_{k=i+1}^{L} Z_{1k}^{(2)} \end{aligned} \tag{2.44}$$

and the probability density function of the random variable Z_1 is obtained as

$$f_{Z_1}(z_1|i) = \left[f_{Z_{1k}^{(1)}}(z_{1k}^{(1)}) \right]^{\otimes i} \otimes \left[f_{Z_{1k}^{(2)}}(z_{1k}^{(2)}) \right]^{\otimes (L-i)} \tag{2.45}$$

It is difficult to evaluate equation (2.45), so we leave this problem for numerical analysis.

Now the probability of symbol error of the self-normalized FFH/MFSK receiver is

$$P_s(i) = Pr(U_1 < U_2 \bigcup U_1 < U_3 \bigcup \cdots \bigcup U_1 < U_M | i) \tag{2.46}$$

For $M > 2$, an analytic solution of (2.46) is not possible because the random variables $U_1, U_2, U_3, \cdots U_{M-1}$ are not independent. Hence, we use the union bound to obtain an upper bound on system performance. From (2.46)

$$P_s(i) \;\leq\; Pr(U_1 < U_2|i) \;+\; Pr(U_1 < U_3|i) \;+\cdots+\; Pr(U_1 < U_M|i) \qquad (2.47)$$

Since we assume the sample statistics of all noise only branches are the same, then (2.47) becomes

$$P_s(i) \;\leq\; (M-1)\; Pr(U_1 < U_m|i) \qquad (2.48)$$

where $m = 2, 3, 4, \cdots, M$. Substituting (2.32) into (2.48), we have

$$P_s(i) \leq (M-1) Pr(Z_1 < 0|i) \qquad (2.49)$$

Since

$$Pr(Z_1 < 0) \;=\; \int_{-L}^{0} f_{Z_1}(z_1|i) dz_1 \qquad (2.50)$$

then from (2.49)

$$P_s(i) \;\leq\; (M-1) \int_{-L}^{0} f_{Z_1}(z_1|i) dz_1 \qquad (2.51)$$

The probability of symbol error is related to the probability of bit error by [Ref. 5]

$$\begin{aligned} P_b(i) \;&=\; P_s(i) \times \frac{2^{k-1}}{2^k - 1}\,, k \;=\; log_2 M \\ &=\; P_s(i) \times \frac{M}{2(M-1)} \end{aligned} \qquad (2.52)$$

Substituting (2.48) into (2.52), we get

$$P_b(i) \;\leq\; \frac{M}{2} \int_{-L}^{0} f_{Z_1}(z_1|i) dz_1 \qquad (2.53)$$

Equation (2.53) must be evaluated numerically.

III. NUMERICAL ANALYSIS

Results of the probability of error analysis for both the conventional receiver and the self-nomalization receiver are obtained for $L = 1$ to 6 hops per bit for several signal-to-noise ratios and direct-to-diffuse component ratios as a function of signal-to-interference ratio to provide an overall comparison of channel and interference effects on system performance for both receivers. In all cases the evaluation of either (2.22) or (2.41) is accomplished via a numerical integration routine. We consider $L + 1$ different cases for a diversity of L hops. For example, for $L = 1$ the two cases to be considered are one non-jammed hop and one jammed hop. For $L = 2$ there are three cases: neither hop jammed, one non-jammed hop and one jammed hop, and both hops jammed. Additionally, the effect of the jamming ratio γ on system performance is examined. We can relate the signal-to-noise ratio to the signal energy density-to-noise power ratio for a jammed hop as

$$\frac{\alpha_k^2 + 2\sigma^2}{\sigma_{kj}^2} = \frac{E_b}{N_0 + N_j/\gamma} \frac{B}{R_s} \frac{\log_2 M}{L} \tag{3.1}$$

where

α_k^2 : direct signal power component

$2\sigma^2$: diffuse signal power component

$\alpha_k^2 + 2\sigma^2$: total signal power

σ_{kj}^2 : thermal noise plus jamming noise power

E_b : average signal bit energy density

N_0 : thermal noise power spectral density

N_j : jamming noise power spectral density

γ : jamming ratio

B	: IF bandwidth
R_s	: symbol rate
M	: number of symbols in alphabet
L	: number of hops in a symbol

For hops with no partial-band jamming

$$\frac{\alpha_k^2 + 2\sigma^2}{\sigma_k^2} = \frac{E_b}{N_0}\frac{B}{R_s}\frac{\log_2 M}{L} \tag{3.2}$$

where σ_k^2 is the thermal noise power. Assuming the IF bandwidth (B) is equal to the symbol rate, we can rewrite (3.1) and (3.2)

$$\frac{\alpha_k^2 + 2\sigma^2}{\sigma_{kj}^2} = \frac{E_b}{N_0 + N_j/\gamma}\frac{\log_2 M}{L} \tag{3.3}$$

and

$$\frac{\alpha_k^2 + 2\sigma^2}{\sigma_k^2} = \frac{E_b}{N_0}\frac{\log_2 M}{L} \tag{3.4}$$

For computational purposed, define

$$\sigma_{kj}^2 = 1 \tag{3.5}$$

$$\frac{E_b}{N_0} = \eta \tag{3.6}$$

$$\frac{E_b}{N_j} = \nu \tag{3.7}$$

$$\frac{\alpha_k^2}{2\sigma^2} = \psi \tag{3.8}$$

$$\log_2 M = \omega \tag{3.9}$$

Substituting (3.5) through (3.9) into (3.3) and (3.4), we have

$$2\sigma^2 = \frac{\eta\gamma\nu}{(\eta+\gamma\nu)(\psi+1)}\frac{\omega}{L} \tag{3.10}$$

$$\alpha_k^2 = \frac{\eta\gamma\nu\psi}{(\eta+\gamma\nu)(\psi+1)}\frac{\omega}{L} \tag{3.11}$$

$$\sigma_k^2 = \frac{\gamma\nu}{(\eta+\gamma\nu)} \tag{3.12}$$

Substituting (3.10), (3.11), and (3.12) into the probability density function of the conventional receiver and the probability density function of self-normalization receiver and using an fast Fourier transform (FFT) routine with a sequence length of N=1024, numerical sequences of the probability density functions are generated for various signal-to-noise ratios of the jammed and non-jammed cases.

A. Conventional Receiver

Except for a single convolution, we have obtained analytic solution for the probability density function of the decision statistic for the conventional FFH/MFSK detector. Unfortunately, the modified Bessel function terms make it difficult to obtain a complete analytic solution for the general case. Consequently, we use an FFT to perform the convolution numerically. Even though we have an analytic expression for the probability density function of the noise branch statistic (the last part of (2.22)), we use numerical methods to correspond with the signal statistic part which must be evaluated numerically. Hence, we use different equations for the numerical procedure. Equation (2.16) can be rewritten

$$f_{X_m}(x_m) = f_{X_m}^{[1]}(x_m) \bigotimes f_{X_m}^{[2]}(x_m) \tag{3.13}$$

where

$$f_{X_m}^{[1]}(x_m) = \left[\frac{1}{2\sigma_{kj}^2}\, e^{-\frac{x_m}{2\sigma_{kj}^2}}\right]^{\otimes i} \tag{3.14}$$

$$f_{X_m}^{[2]}(x_m) = \left[\frac{1}{2\sigma_{k}^2}\, e^{-\frac{x_m}{2\sigma_{k}^2}}\right]^{\otimes (L-i)} \tag{3.15}$$

Using the Laplace transform, we get

$$F_{X_m}^{[1]}(s) = \left[\frac{\beta_k^{(1)}}{s+\beta_k^{(1)}}\right]^{i} \tag{3.16}$$

$$F_{X_m}^{[2]}(s) = \left[\frac{\beta_k^{(2)}}{s+\beta_k^{(2)}}\right]^{L-i} \tag{3.17}$$

where

$$\beta_k^{(1)} = \frac{1}{2\sigma_{kj}^2} \tag{3.18}$$

$$\beta_k^{(2)} = \frac{1}{2\sigma_k^2} \tag{3.19}$$

The inverse Laplace transform of (3.16) and (3.17) yields

$$f_{X_m}^{[1]}(x_m) = \left(\frac{1}{2\sigma_{kj}^2}\right)^i \frac{x_m^{i-1}}{(i-1)!} e^{-\frac{x_m}{2\sigma_{kj}^2}} \tag{3.20}$$

$$f_{X_m}^{[2]}(x_m) = \left(\frac{1}{2\sigma_k^2}\right)^{L-i} \frac{x_m^{L-i-1}}{(L-i-1)!} e^{-\frac{x_m}{2\sigma_k^2}} \tag{3.21}$$

Now we have a different form of the probability density function of the random variable X_m

$$\begin{aligned} f_{X_m}(x_m) = & \left[\left(\frac{1}{2\sigma_{kj}^2}\right)^i \frac{x_m^{i-1}}{(i-1)!} e^{-\frac{x_m}{2\sigma_{kj}^2}}\right] \\ & \otimes \left[\left(\frac{1}{2\sigma_k^2}\right)^{L-i} \frac{x_m^{L-i-1}}{(L-i-1)!} e^{-\frac{x_m}{2\sigma_k^2}}\right] \end{aligned} \tag{3.22}$$

Instead of (2.21) we substitute (3.22) into (2.22) and analyze (2.22) numerically because we need a numerical sequence of this formula to coincide with the numerical sequence obtained for (2.11) The length of the basic sequence is N=1024, and prior to convolution it is zero-padded to create a sequence of total length N=2048 since the sequence is to be convolved one time.

Numerical problems are encountered since as the value of the variable x_1 increases the exponential terms approach zero and the modifiied Bessel function terms approach infinity. Hence, the multiplication of the exponential term and the modified Bessel function term is invalid when the argument of the exponent is less than -745 or the argument of the modified Bessel function is greater than 705 when 386Matlab is used. For example, in the situation described above, the value of exp(-745) is 4.9470e-324, but the value of exp(-746) is set to 0 by the computer. The value of

Bessel(705),where the 386 MATLAB function for the modified Bessel function $I_0(\bullet)$ is Bessel($\bullet$), is 2.2621e+304 and Bessel(706) is not defined. Even though the product of the two is either zero or an invalid number, the true value of a multiplication is a valid number. When this situation is encountered, the program evaluates the situation and uses either zero or one (because the cumulative probability density function approaches one as the value of the variable increases).

When two probability density functions are convolved by an FFT, if the valid range of one probability density function is much wider than the range of the other, the result is almost the same as the probability density function that has wider range. We must check for this situation whenever the parameters of the equation change while the program is running, skip the numerical convolution steps, and instead use the probability density function that has wider range as the convolution output.

B. Self-Normalization Receiver

The probability density function of self-normalization receiver (2.41) is relatively complicated and we must use an L-fold convolution to evaluate (2.44); an i-fold convolution for jammed hops, an $(L-i)$-fold convolution for nonjammed hops, and each result must be convolved. To implement the convolution numerically, the numerical sequences of the probability density function of the self-normalization receiver must be zero-padded to a total length of $N \times L$ before the FFT is performed in order to preserve the desired linear convolution of (2.44). The transformed sequences are multiplied point by point to produce the desired seqence of

$$F_{Z_1}(s) = \left[F^{(1)}_{Z_{1k}}(s)\right]^i \times \left[F^{(2)}_{Z_{1k}}(s)\right]^{L-i} \tag{3.23}$$

where F is the Fourier transform of f. This result is then inverse fast Fourier transformed to yield the desired sequence for $f_{Z_1}(z_1)$. The procedures for L=3 and L=5 are similar to that for L=4 and L=6, respectively, because the length of the sequence for the FFT should be a power of two, and the sequences should be zero-padded to N=4096 (2^{12}) for L=3 and L=4 and to N=8192 (2^{13}) for L=5 and L=6.

Equation (2.42) has many summations inside which must be carefully handled since numerical errors are likely when the value of the variable z_{1k} approaches one When z_{1k} approaches unity, the term $1/(1-z_{1k})$ approaches infinity. Therefore, we must group the terms properly to minimize numerical errors. Equation (2.42) is rewritten

$$\begin{aligned}
g(z_{1k}) \;=\; & \frac{z_{1k}(1-z_{1k})^{M-2}(1+\xi_k)^{M-2}}{(1+\xi_k(1-z_{1k}))^{M-1}(1+z_{1k})} \exp\left[\frac{-\rho_k(1-z_{1k})}{1+\xi_k(1-z_{1k})}\right] \\
& \times \sum_{m=0}^{M-2} \frac{(M-2)(M-2)!}{(M-2-m)!(m!)^2} \left[\frac{\rho_k z_{1k}}{(1+\xi_k(1-z_{1k}))(1+\xi_k)}\right]^m \\
& + \sum_{m=0}^{M-3} \exp\left[\frac{-\rho_k(1-z_{1k})}{1+\xi_k(1-z_{1k})}\right] \\
& \times \frac{2z_{1k}^2((1-z_{1k})(1+\xi_k))^{M-3-m}(1+\xi_k)(1+z_{1k})^{m-1}(M-2-m)}{(1+\xi(1-z_{1k}))^{M-1-m}} \\
& \times \sum_{q=0}^{M-2-m} \frac{M-2-m}{(M-2-m-q)!(q!)^2} \left[\frac{\rho_k z_{1k}}{(1+\xi_k(1-z_{1k}))(1+\xi_k)}\right]^q \\
& + \sum_{m=0}^{M-3} \exp\left[\frac{-\rho_k(1-z_{1k})}{1+\xi_k(1-z_{1k})}\right] \frac{((1-z_{1k})(1+\xi_k))^{M-3-m} z_{1k}(1+z_{1k})^m(M-2)}{(1+\xi(1-z_{1k}))^{M-2-m}} \\
& \times \sum_{q=0}^{M-3-m} \frac{M-3-m}{(M-3-m-q)!(q!)^2} \left[\frac{\rho_k z_{1k}}{(1+\xi_k(1-z_{1k}))(1+\xi_k)}\right]^q \qquad (3.24)
\end{aligned}$$

in order to minimize computational errors.

IV. RESULTS

A FFH/MFSK receiver design that demonstrates strong immunity to the partial-band interference is the noise normalization receiver. This receiver requires a measurement of the noise power per hop, but it is not easy to measure the noise power precisely in practical situations. This measurement is not required for the self-normalization receiver; hence, it is easier to implement the self-normalization receiver. As will be seen, partial-band jamming strongly affects the performance of the conventional FFH/MFSK receiver, while the self-normalization FFH/MFSK receiver performs well against partial-band jamming. Probability of bit error analyses for both the conventional system and the self-normalization system assume worst case partial-band jamming. The channel is modeled for a moderately strong direct-to-diffuse signal ratio ($\alpha_k^2/2\sigma^2 = 10$) which we will refer to as Rician, and as a primarily Rayleigh faded channel ($\alpha_k^2/2\sigma^2 = 0.01$). For Rician fading the worst case jamming ratio typically increases as interference power increases. For Rayleigh fading worst case jamming corresponds to broadband jamming (jamming ratio $\gamma = 1$) for all levels of jamming power.

A. Performance of Conventional FFH/MFSK Receiver

Graphs of the probability of bit error, including worst case, as a function of signal-to-jamming ratio for fixed signal-to-thermal noise ratios are obtained for 1 through 6 hops per symbol. These results are obtained by numerically evaluating (2.44) for values of the jamming ratio ranging from 1.0 to 0.001 and retaining worst case performance for each value of E_b/N_j . The results for a Rician fading channel are shown in Fig. 4.1 through Fig. 4.6.

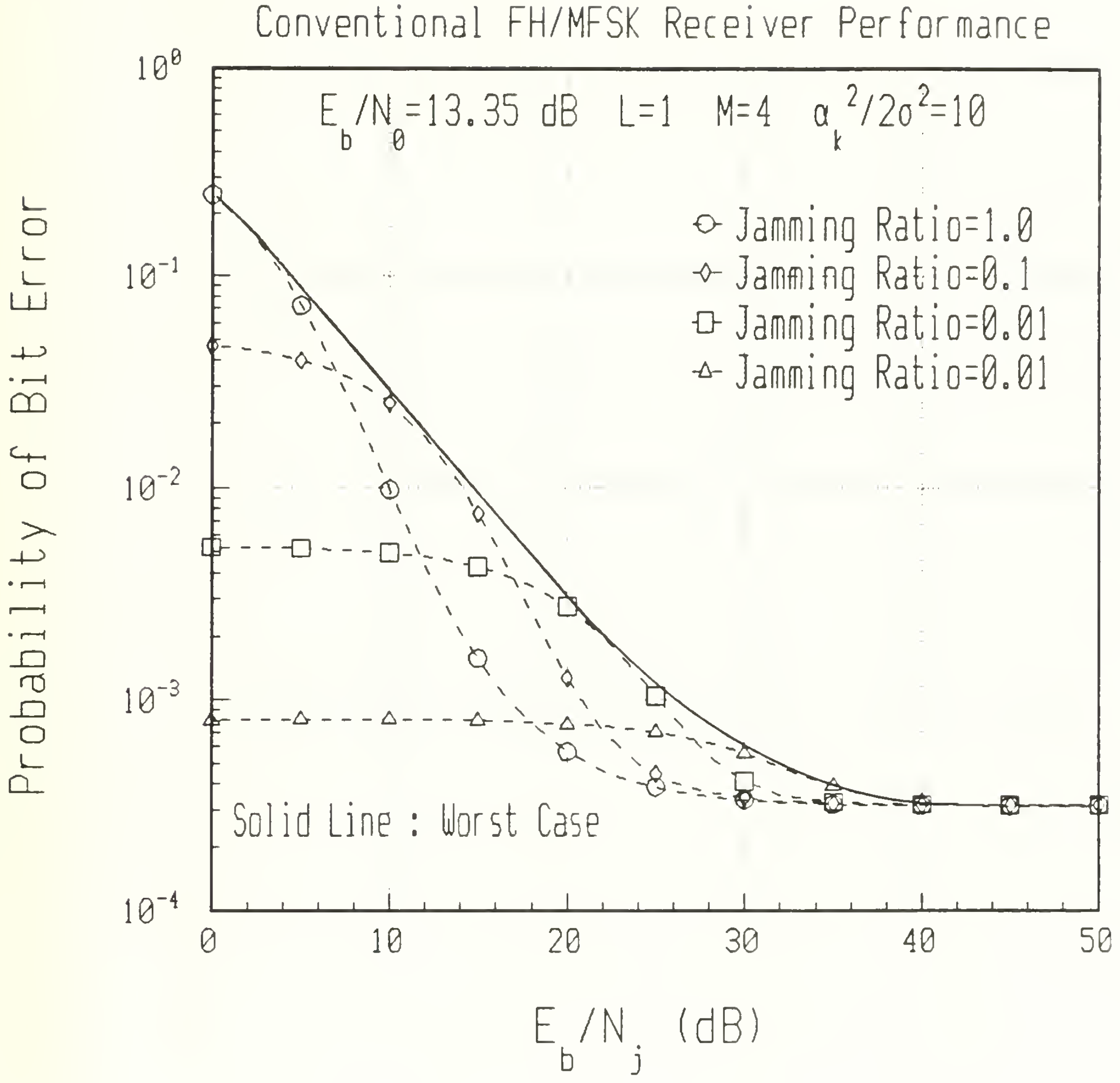


Figure 4.1: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 1, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

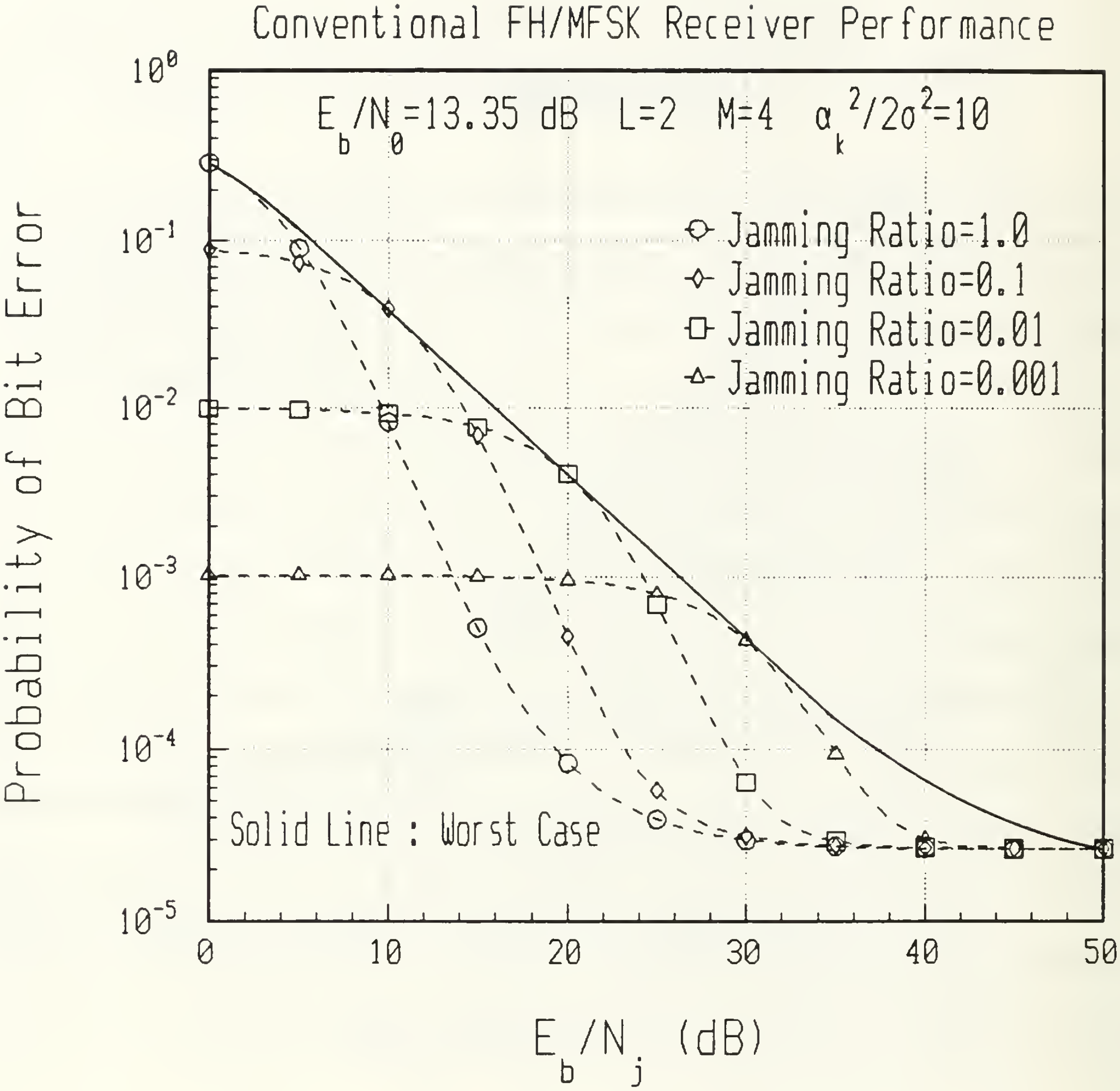


Figure 4.2: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 2, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

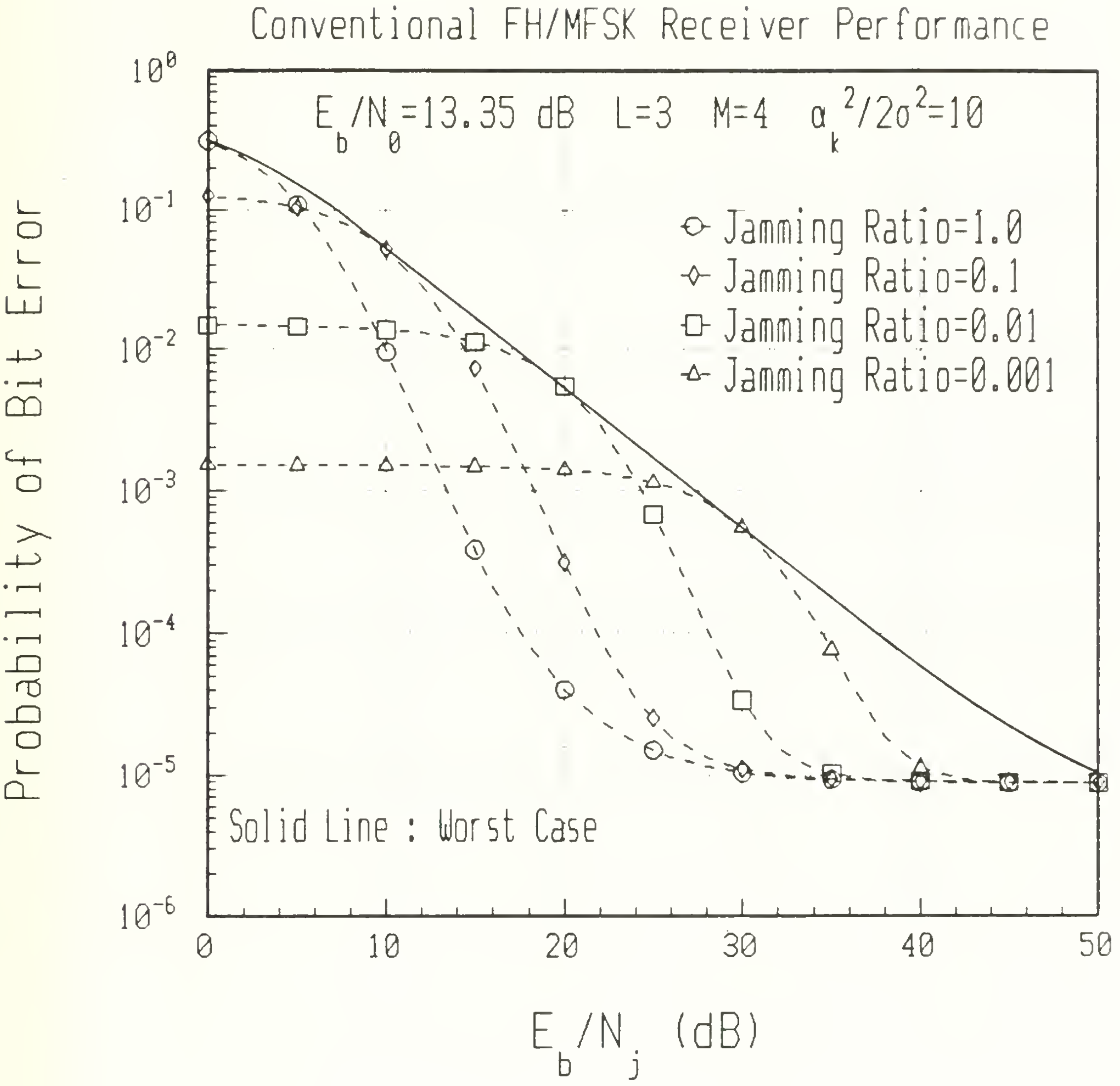


Figure 4.3: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 3, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

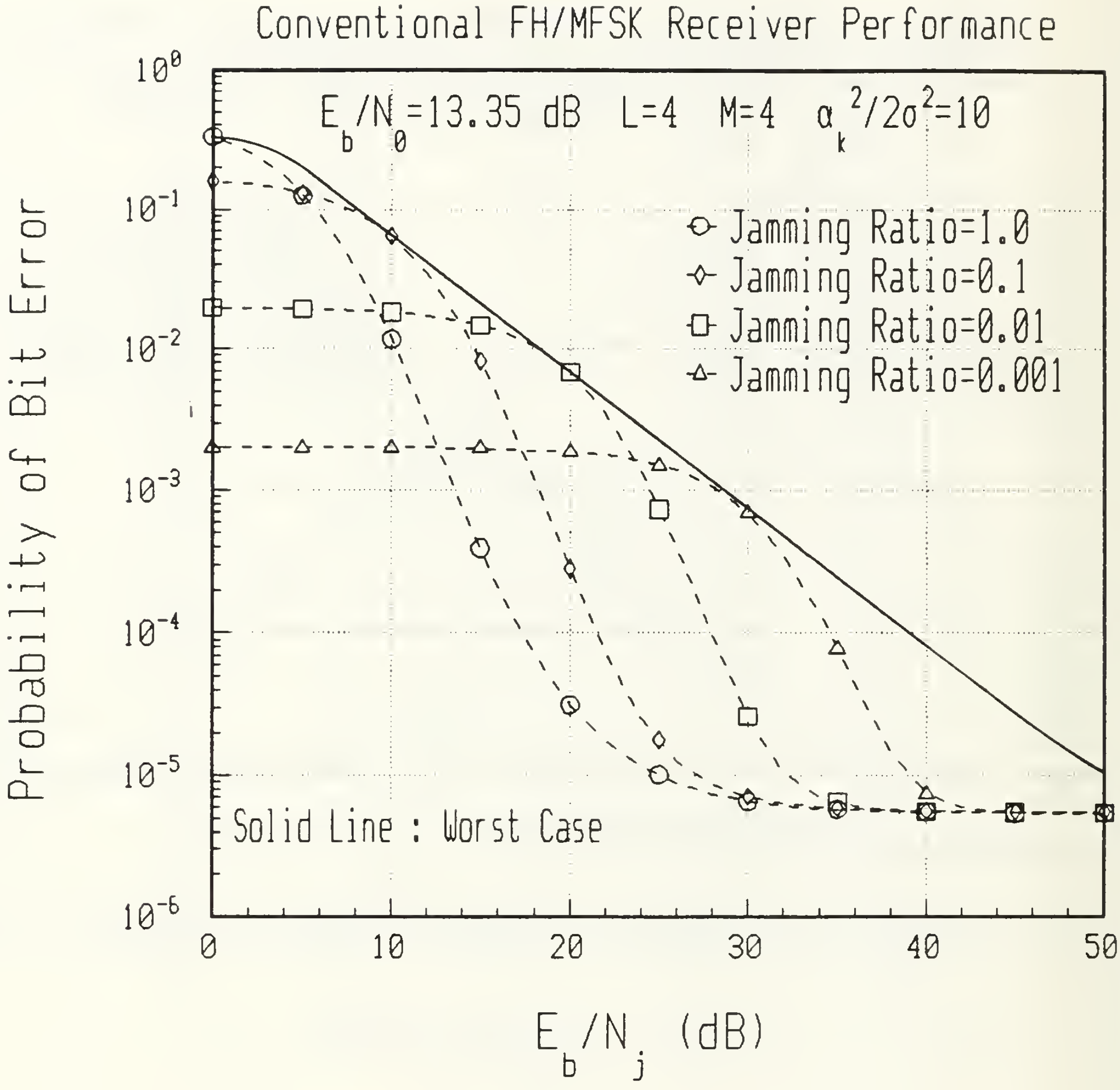


Figure 4.4: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

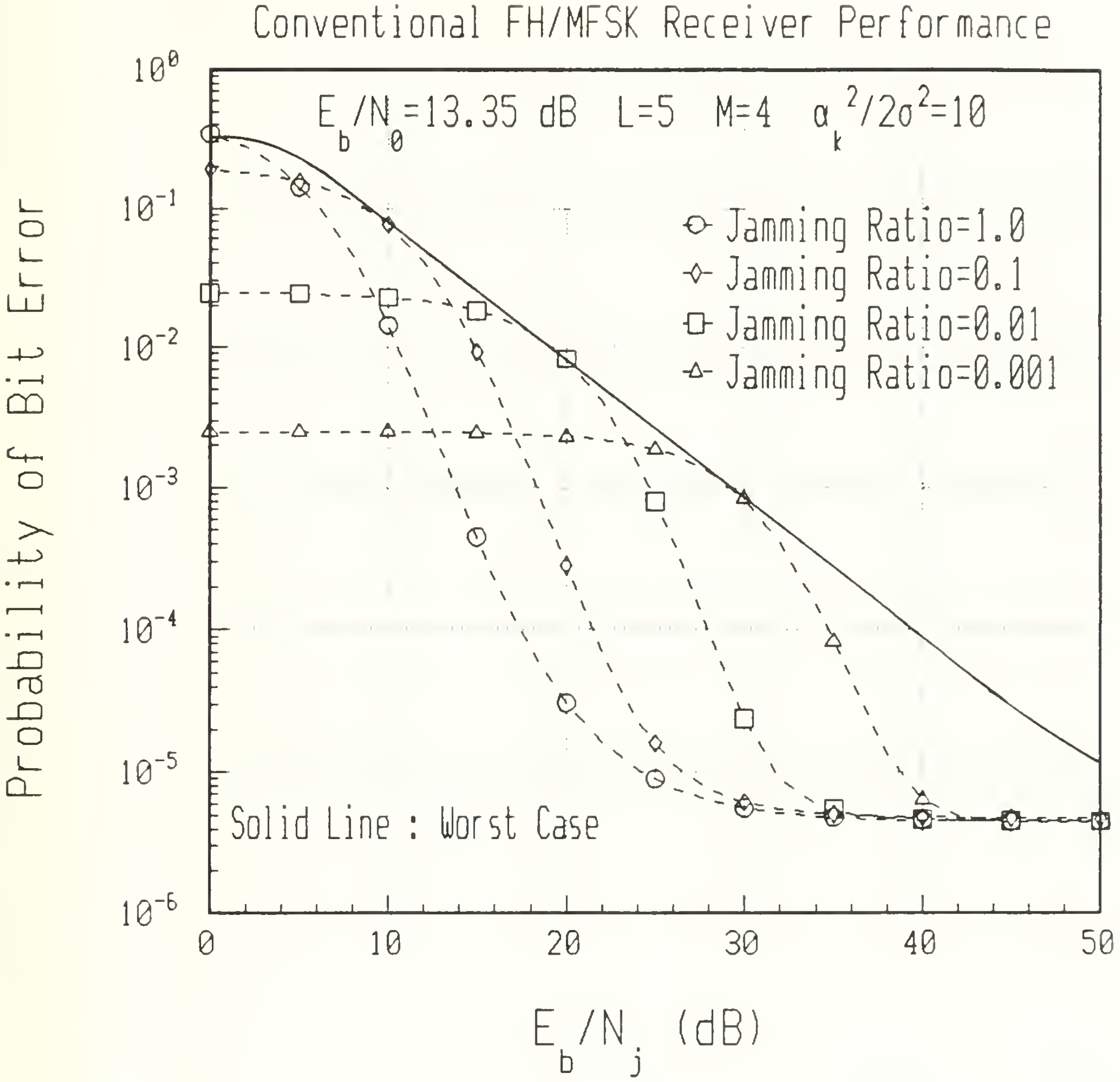


Figure 4.5: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 5, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

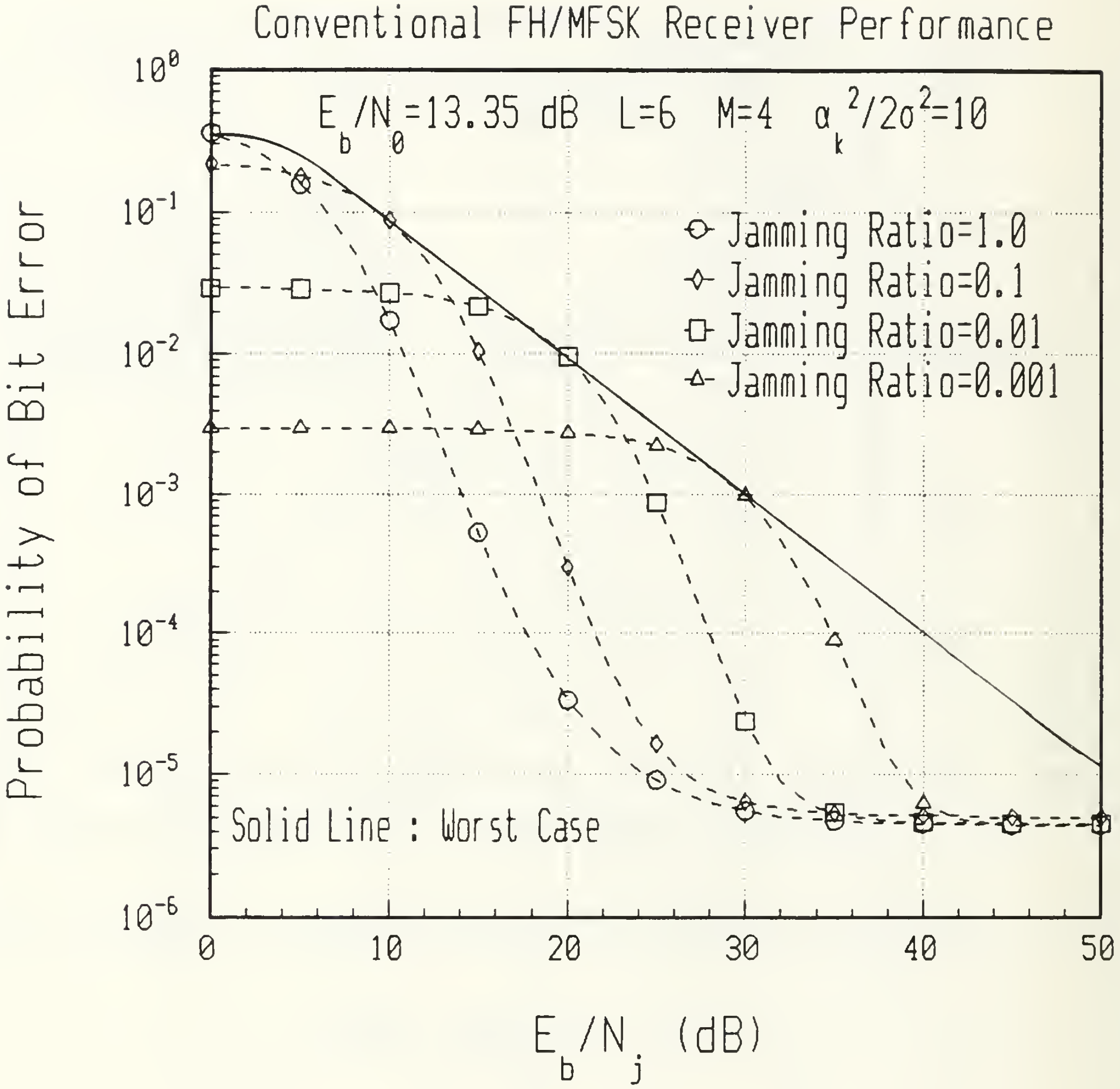


Figure 4.6: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 6, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

The signal-to-noise ratio, direct-to-diffuse ratio, and the modulation order are chosen to enable direct comparison with the results obtained in [Ref. 2]. We see that for a Rician channel that diversity is not effective in reducing the effects of worst case partial-band jamming. Worst case performance is poorer when diversity is used than for no diversity. As diversity increases, only the asymtotic value for high E_b/N_j approaches the performance of the noise-nomalization receiver. The order of modulation has no effect with respect to providing immunity to partial-band interference. This is illustrated in Fig. 4.4 and Fig. 4.7 through Fig. 4.9. Generally both diversity and the order of modulatin are effective in improving system performance for full-band jamming ($\gamma = 1$) as shown Fig. 4.10 and Fig. 4.11.

Generally both diversity and the order of modulation for a Rayleigh fading channel are effective in improveing system performance for full band jamming ($\gamma = 1$) as shown in Fig. 4.10 and Fig. 4.11. However, System performance, shown in Fig. 4.12 through Fig. 4.16, is not improved by either diversity or modulation order when worst case partial-band jamming is assumed. When the signal-to-thermal noise ratio increases, the performance of the conventional receiver for worst case partial-band jamming is unchanged from that of lower signal-to-thermal noise cases except for the asymtotic limit of negligible E_b/N_j.

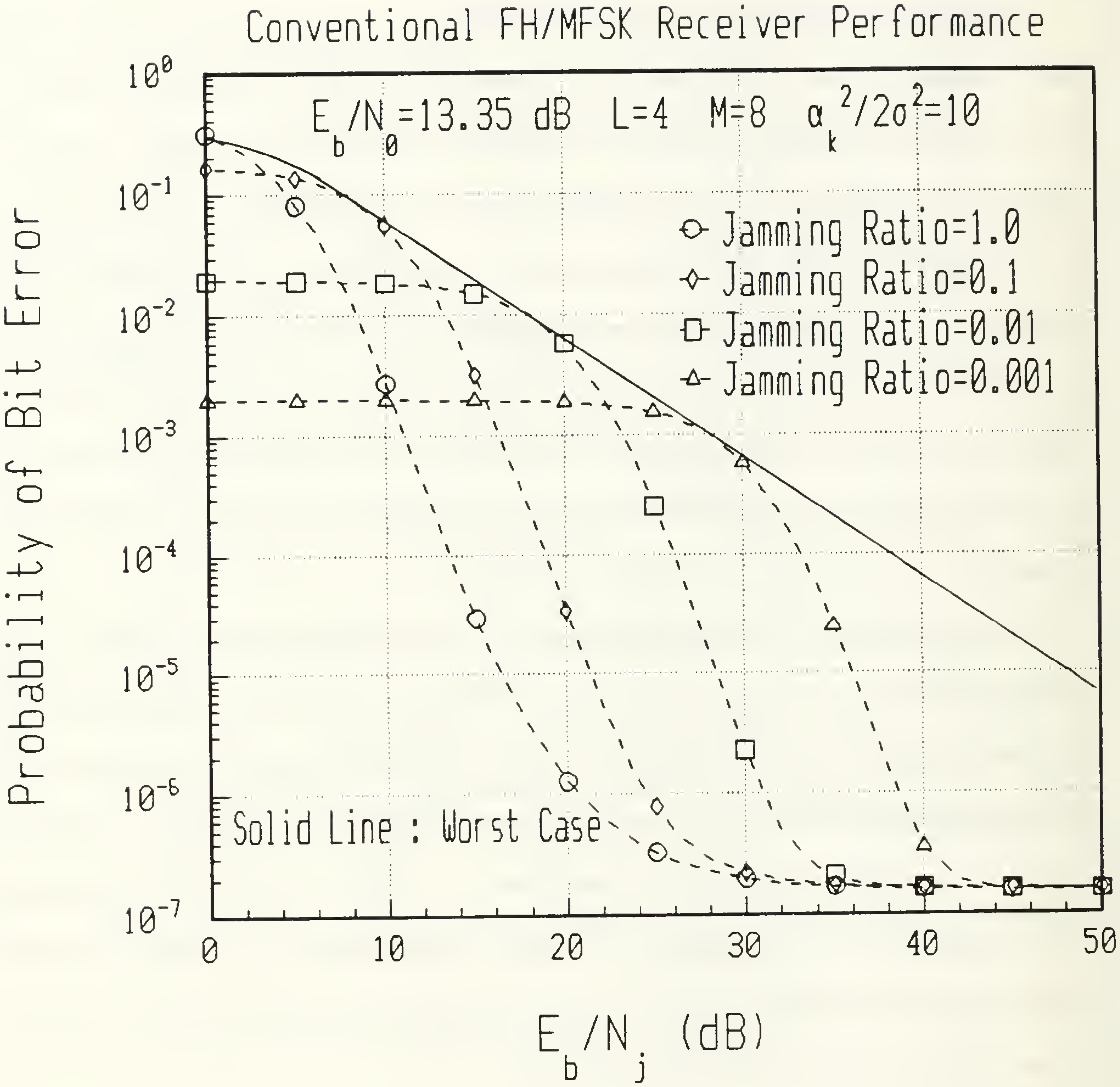


Figure 4.7: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 8 and Direct-to-Diffuse Signal Power Ratio = 10

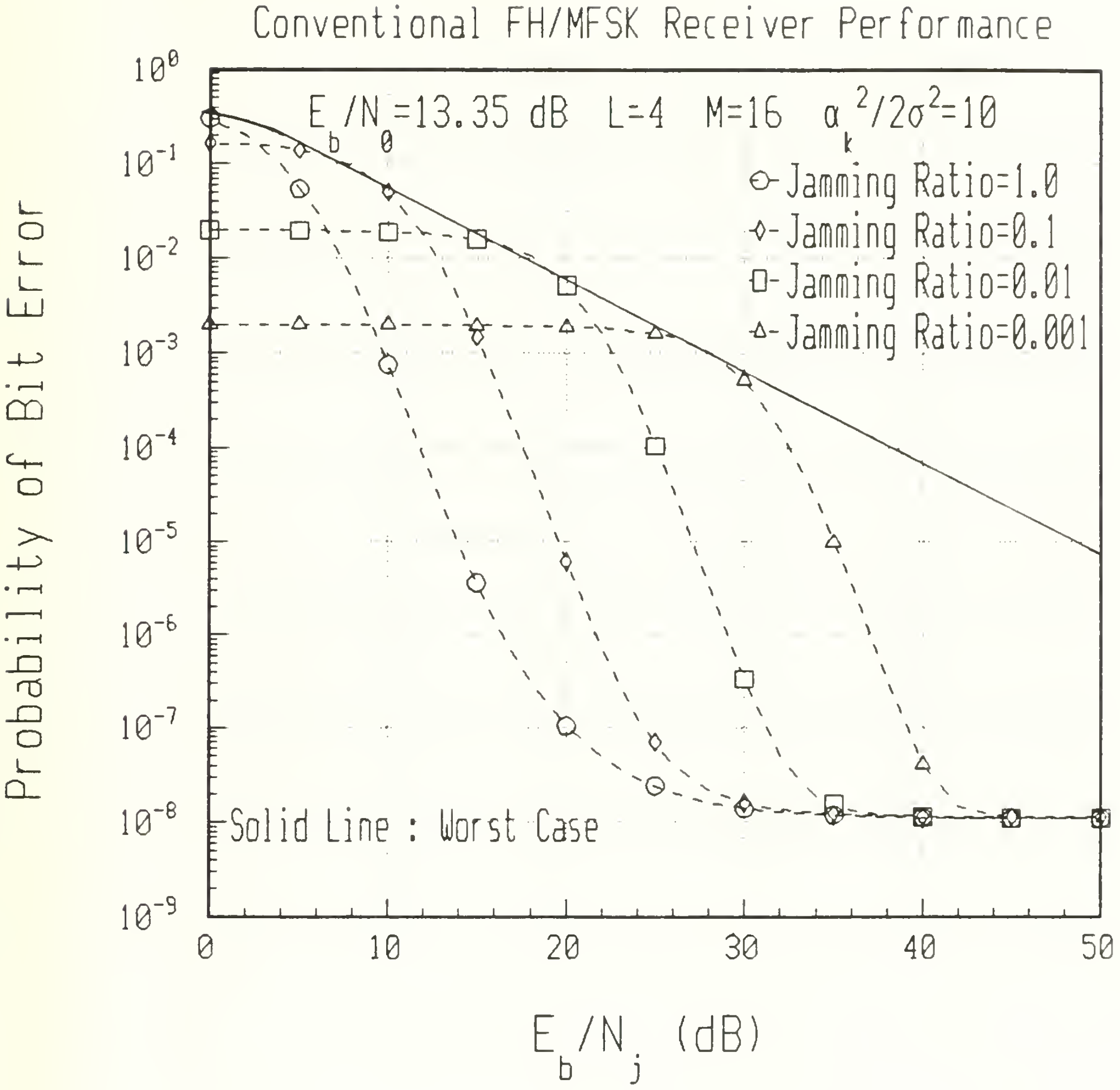


Figure 4.8: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 16 and Direct-to-Diffuse Signal Power Ratio = 10

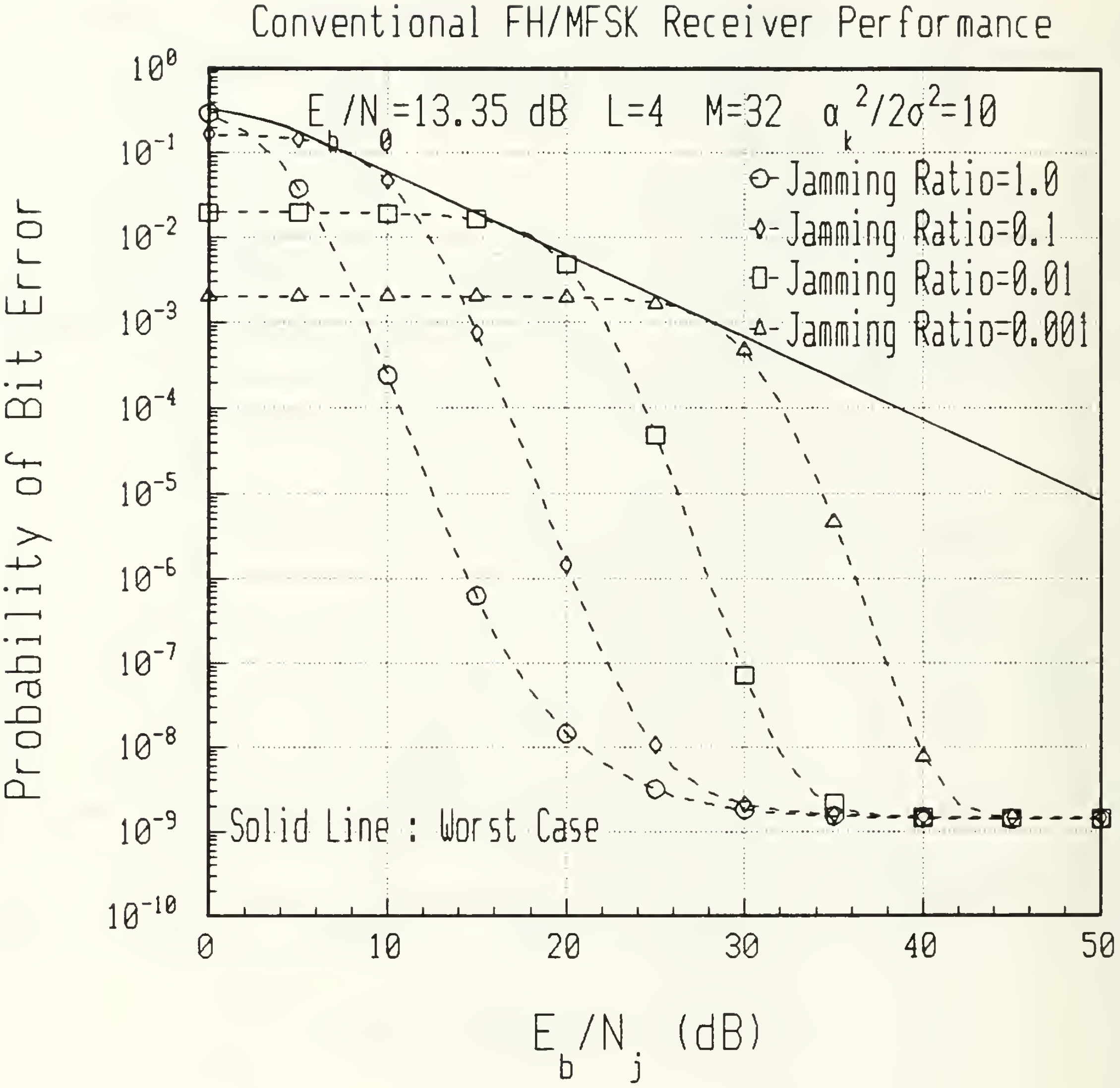


Figure 4.9: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 32 and Direct-to-Diffuse Signal Power Ratio = 10

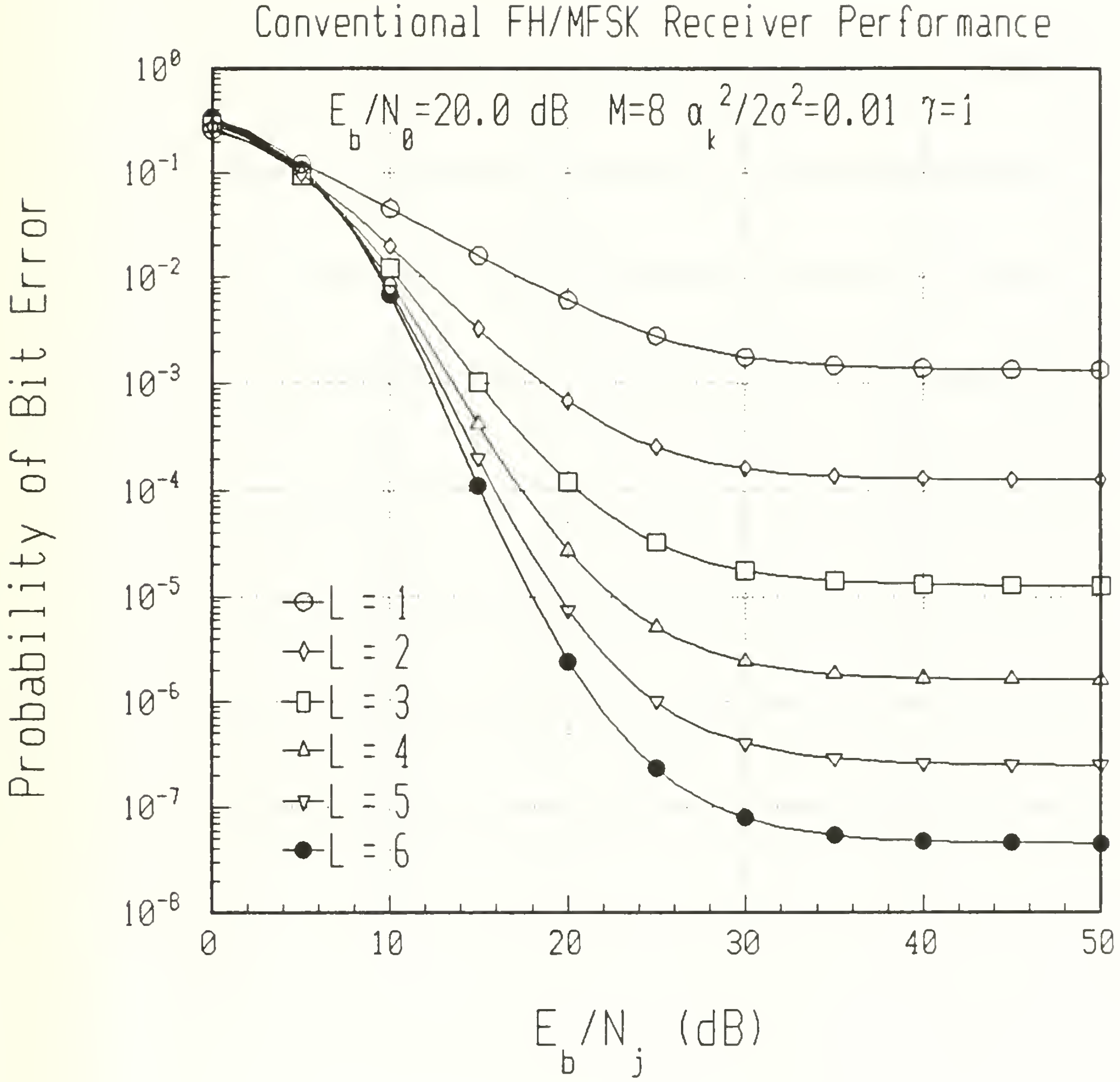


Figure 4.10: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 20.0dB, Jamming Ratio = 1, Order of Modulation = 8 and Direct-to-Diffuse Signal Power Ratio = 0.01

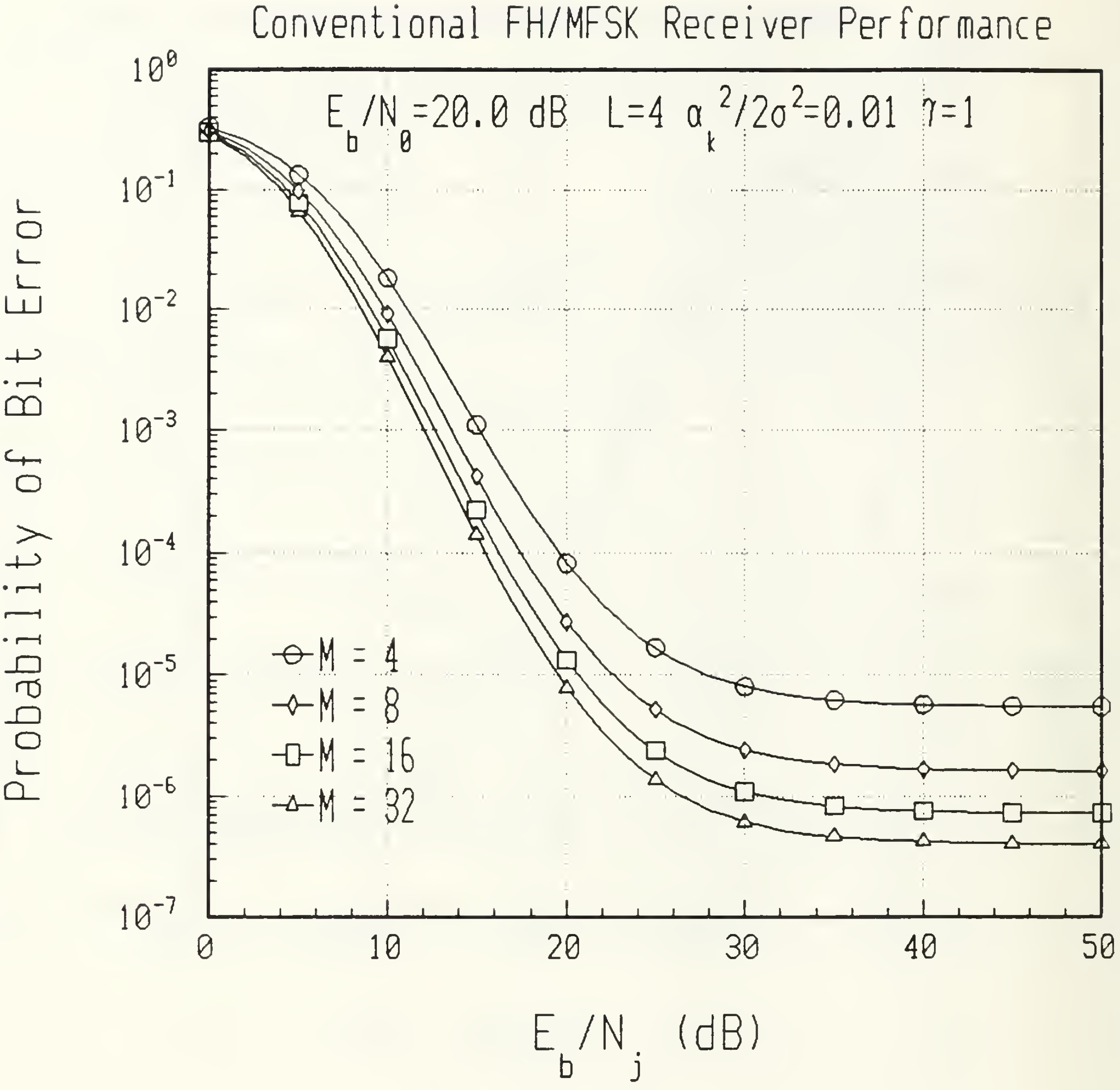


Figure 4.11: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 20.0dB, Hopping Number per Symbol = 4, Jamming Ratio = 1 and Direct-to-Diffuse Signal Power Ratio = 0.01

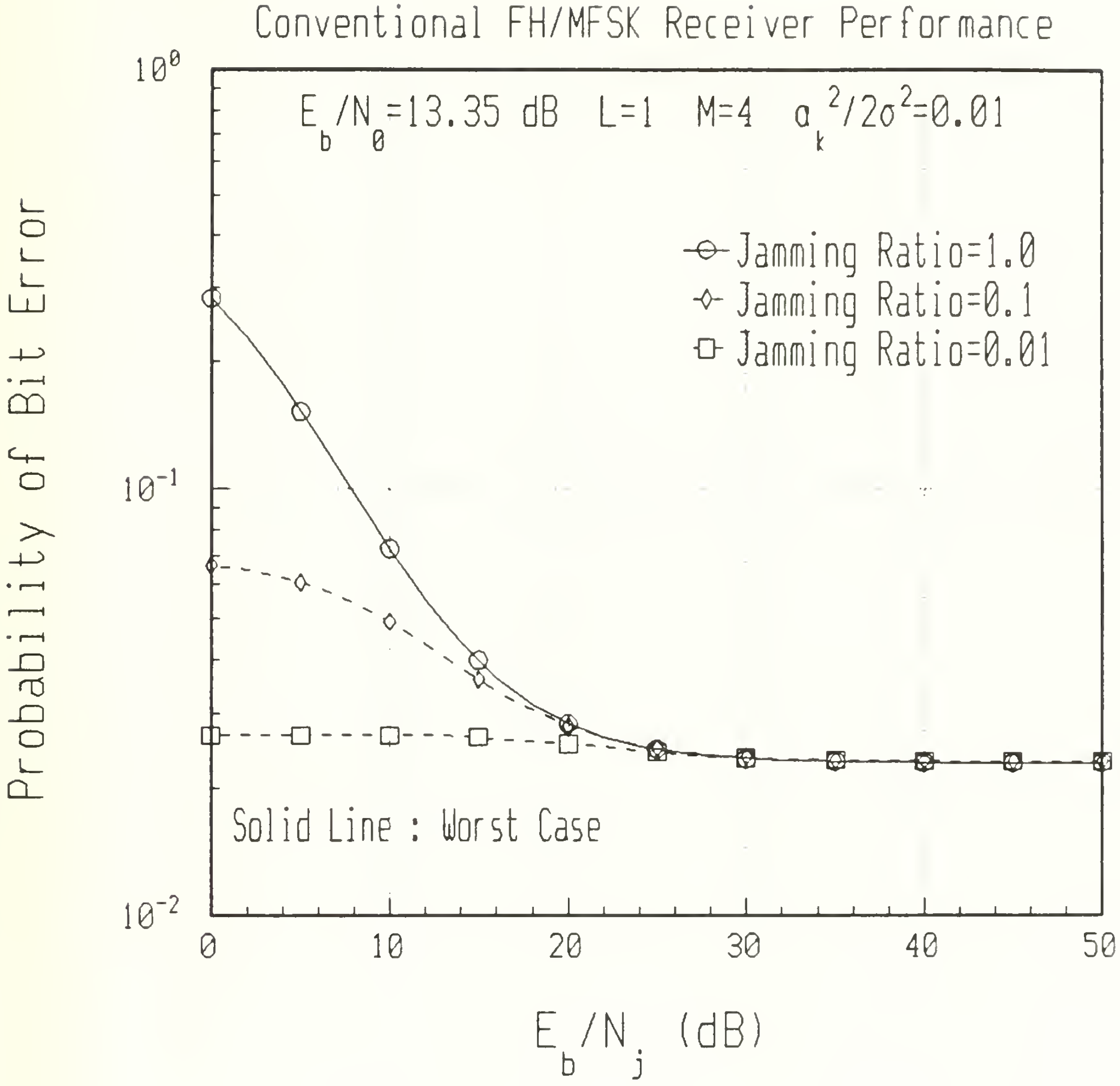


Figure 4.12: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 1, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 0.01

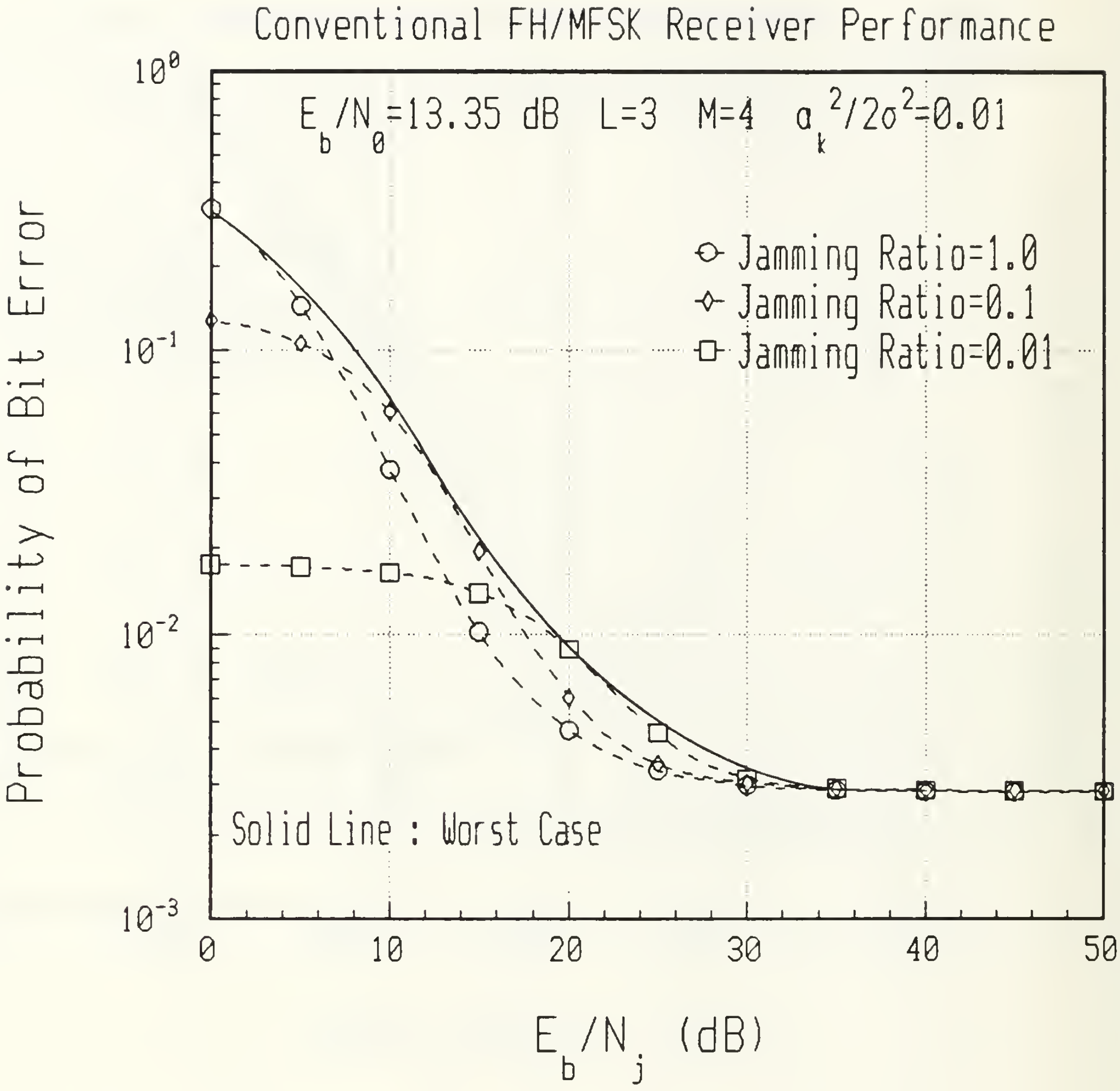


Figure 4.13: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 3, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 0.01

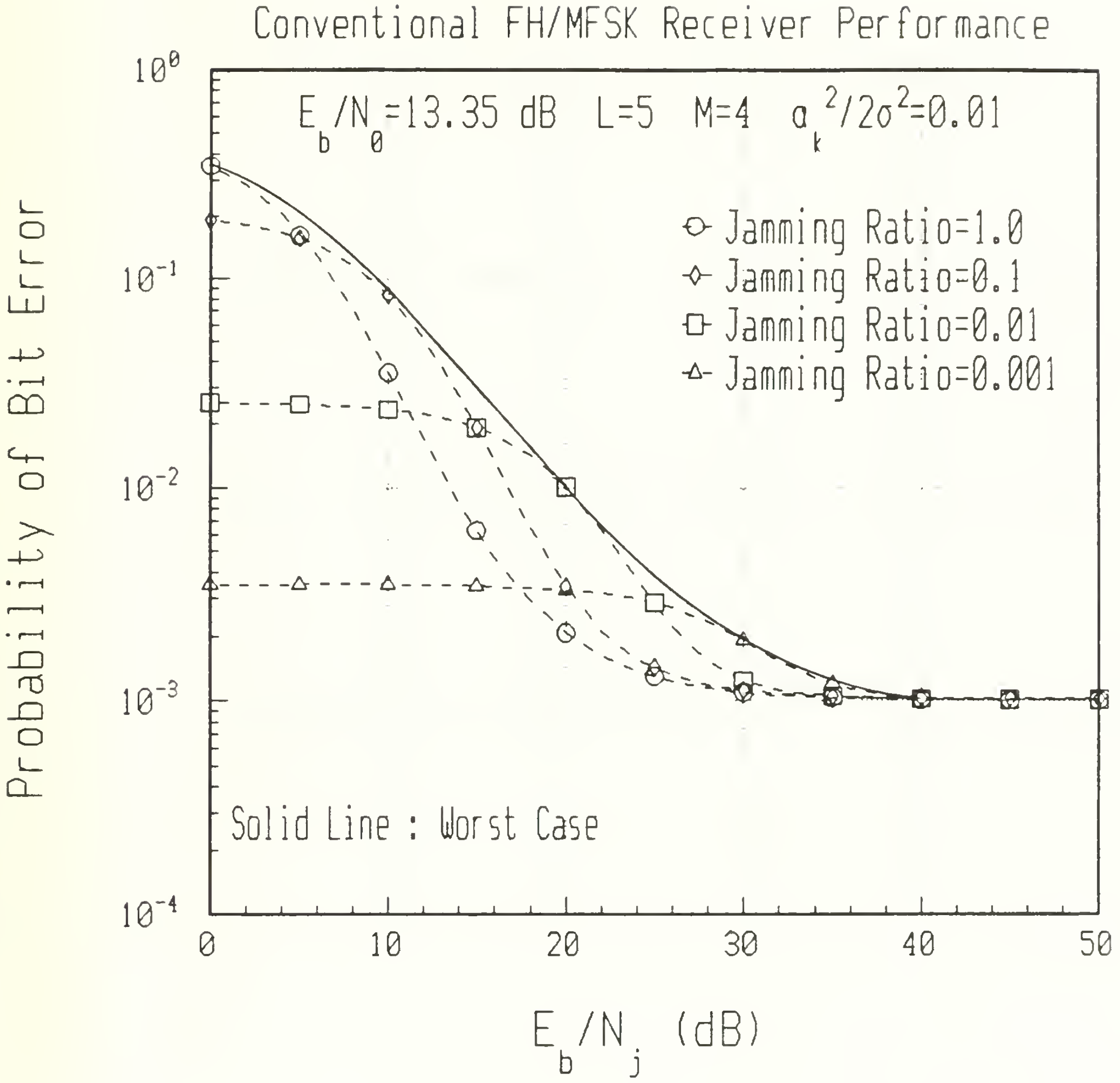


Figure 4.14: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 5, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 0.01

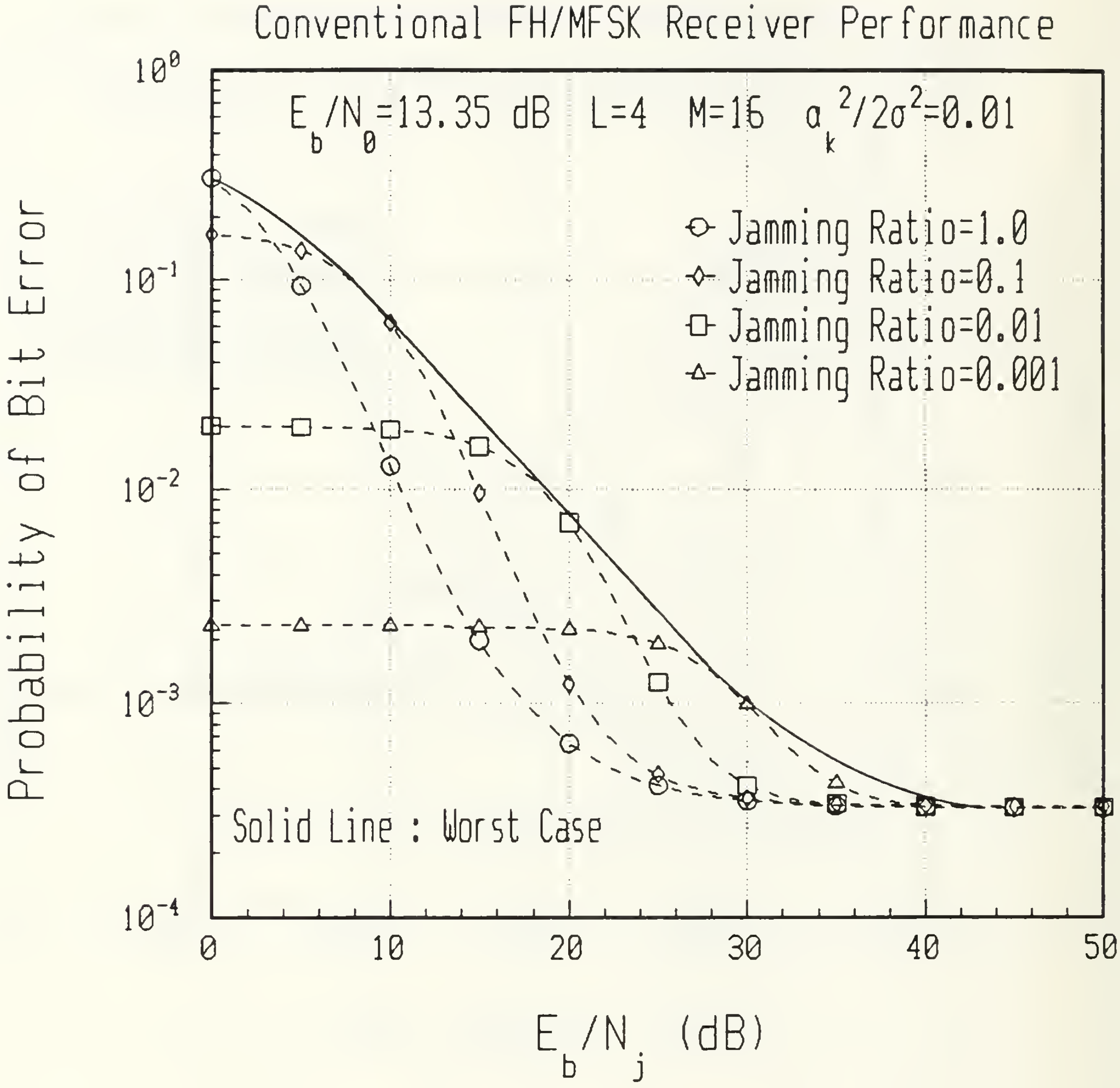


Figure 4.15: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 16 and Direct-to-Diffuse Signal Power Ratio = 0.01

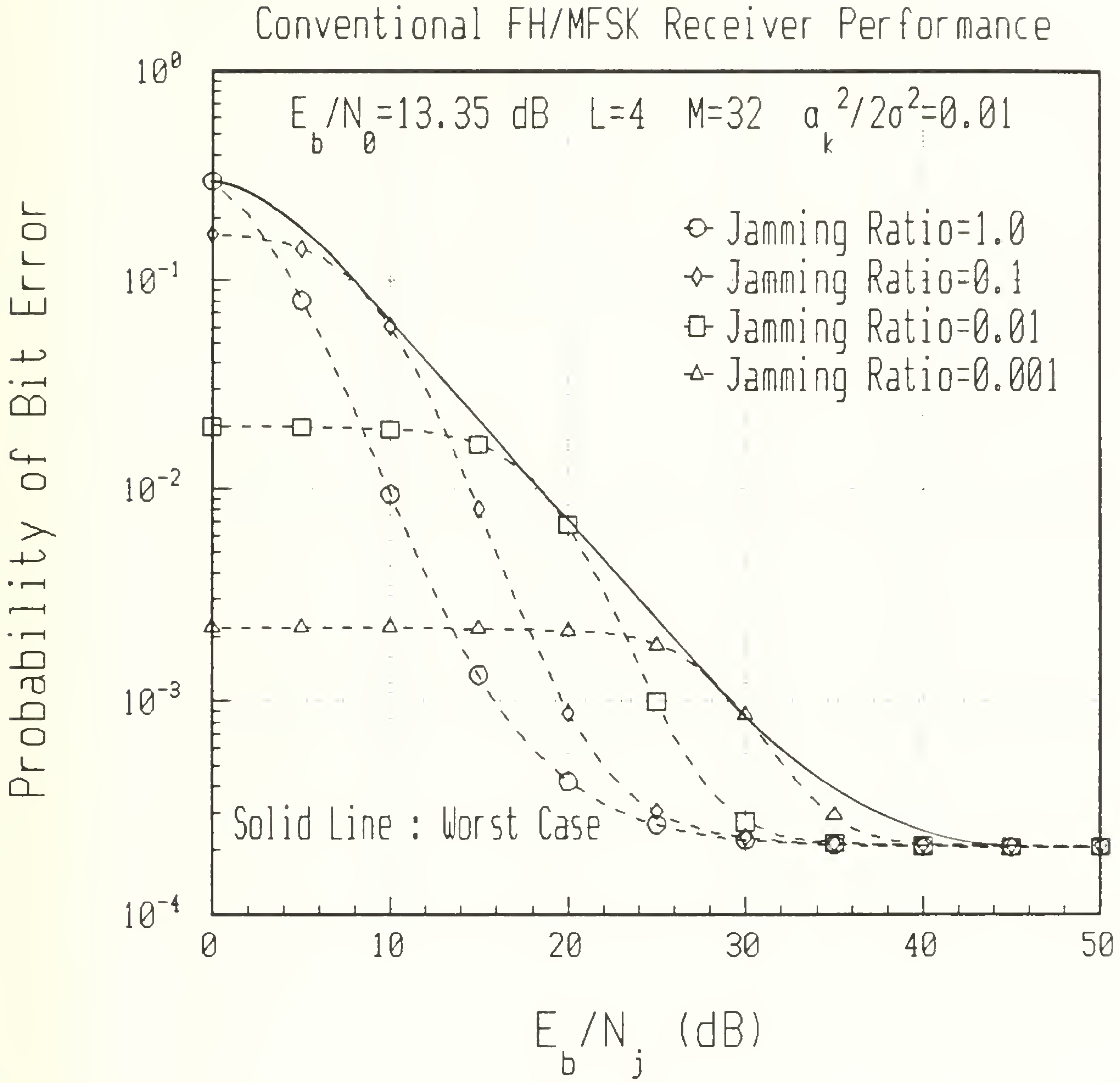


Figure 4.16: The Performance of the Conventional FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 32 and Direct-to-Diffuse Signal Power Ratio = 0.01

B. Performance of Self-Normalization FFH/MFSK Receiver

All parameters used in the graphs for the self-normalization receiver are the same as those used for the conventional receiver to make it easier to compare with each other as well as with the noise-normalization receiver. As we see from Fig. 4.17 through Fig. 4.25, for a Rician fading channel, increasing diversity and increasing modulation order are both effective in minimizing the effects of worst case partial-band interference.

Overall, the performance of the self-normalization receiver is almost the same as that of the noise-normalization receiver. There exists a small difference in the performance of the two receivers. This difference is expected due to the union bound used to obtain the FFH/MFSK self-normalized receiver performance. The union bound has the most pronounced effect on system performance when E_b/N_j is small and the modulation order is high. See, for example, Fig. 4.26 where the probability of bit error for a bit energy-to-jamming noise density ratio of 0 dB with a jamming ratio 1.0 is shown greater than unity. This is not valid for a probability. However, the error performance at low E_b/N_j is not important, and the error due to the union bound compared to the exact value at high E_b/N_j is not significant.

The performance of the self-normalization receiver for a Rayleigh fading channel in worst case partial-band jamming is the same as for full band jamming ($\gamma = 1$) regardless of the parameters as shown in Fig. 4.27 through Fig. 4.29. Generally for a Rayleigh fading channel, diversity is effective in improving the performance of the self-normalization receiver. This is shown in Fig. 4.30. The effect of increasing the modulation order is less significant as shown in Fig. 4.26. As mentioned previously, the union bound makes the simulated performance worse than the true performance.

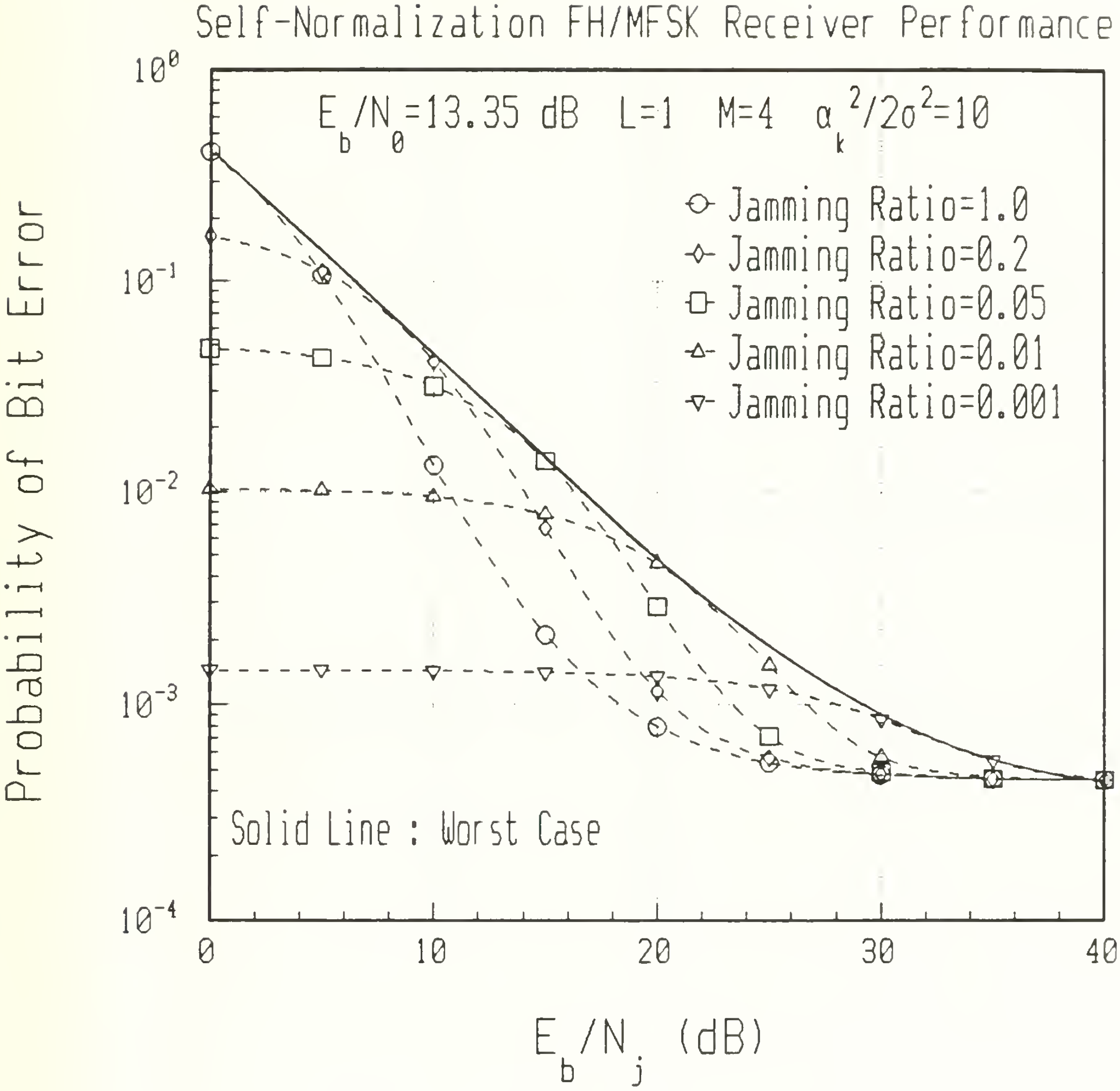


Figure 4.17: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 1, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

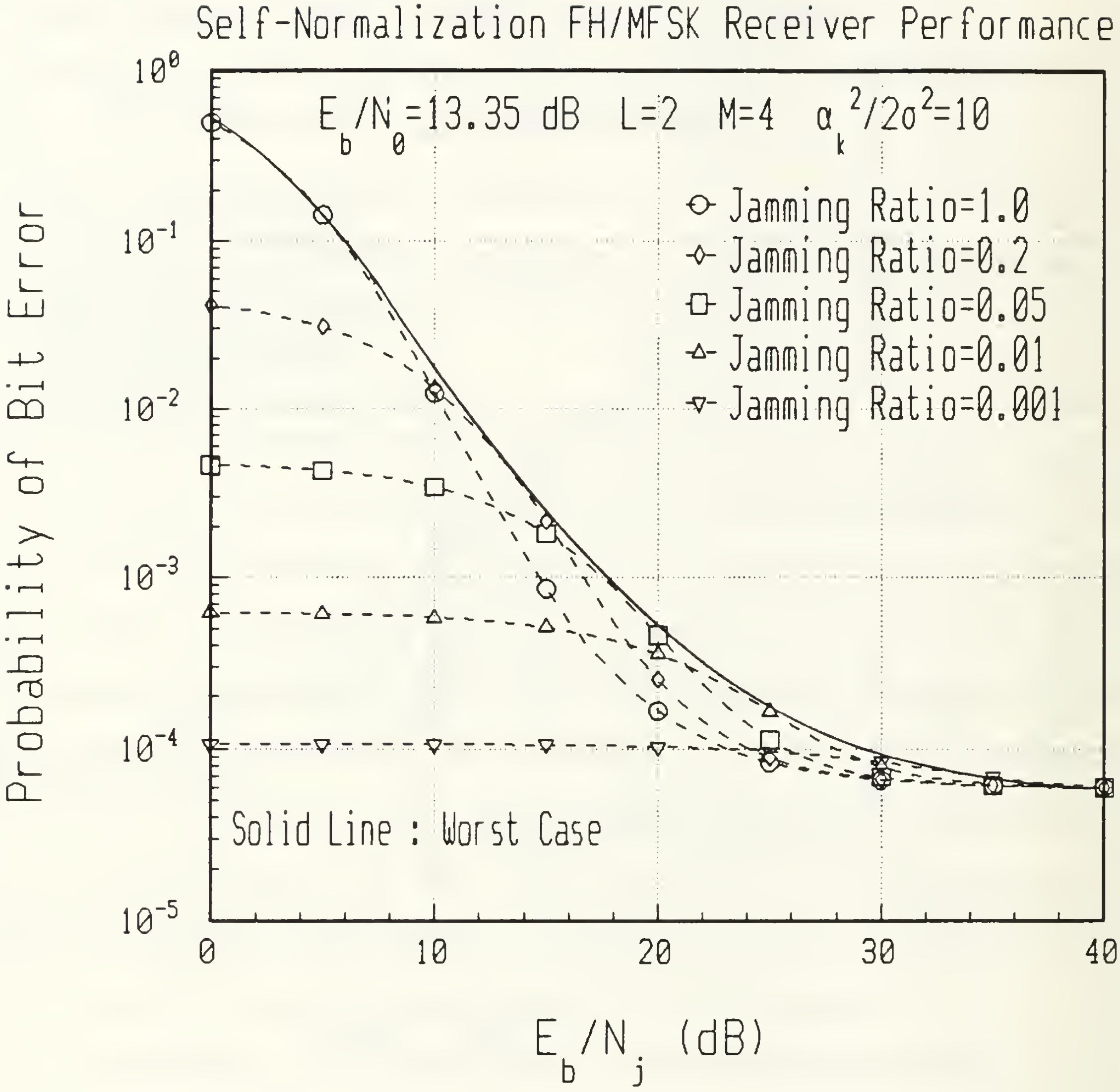


Figure 4.18: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 2, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

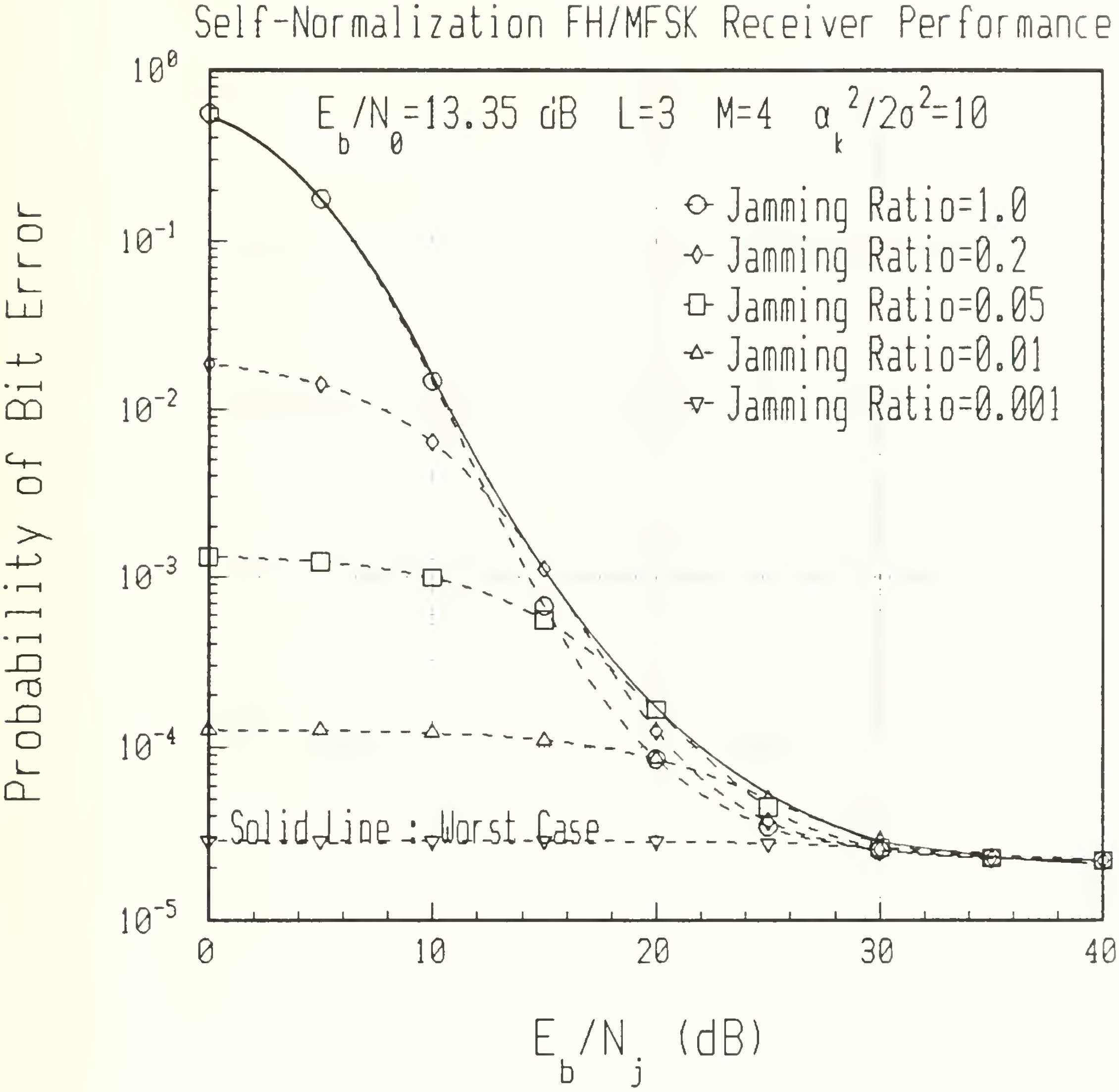


Figure 4.19: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 3, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

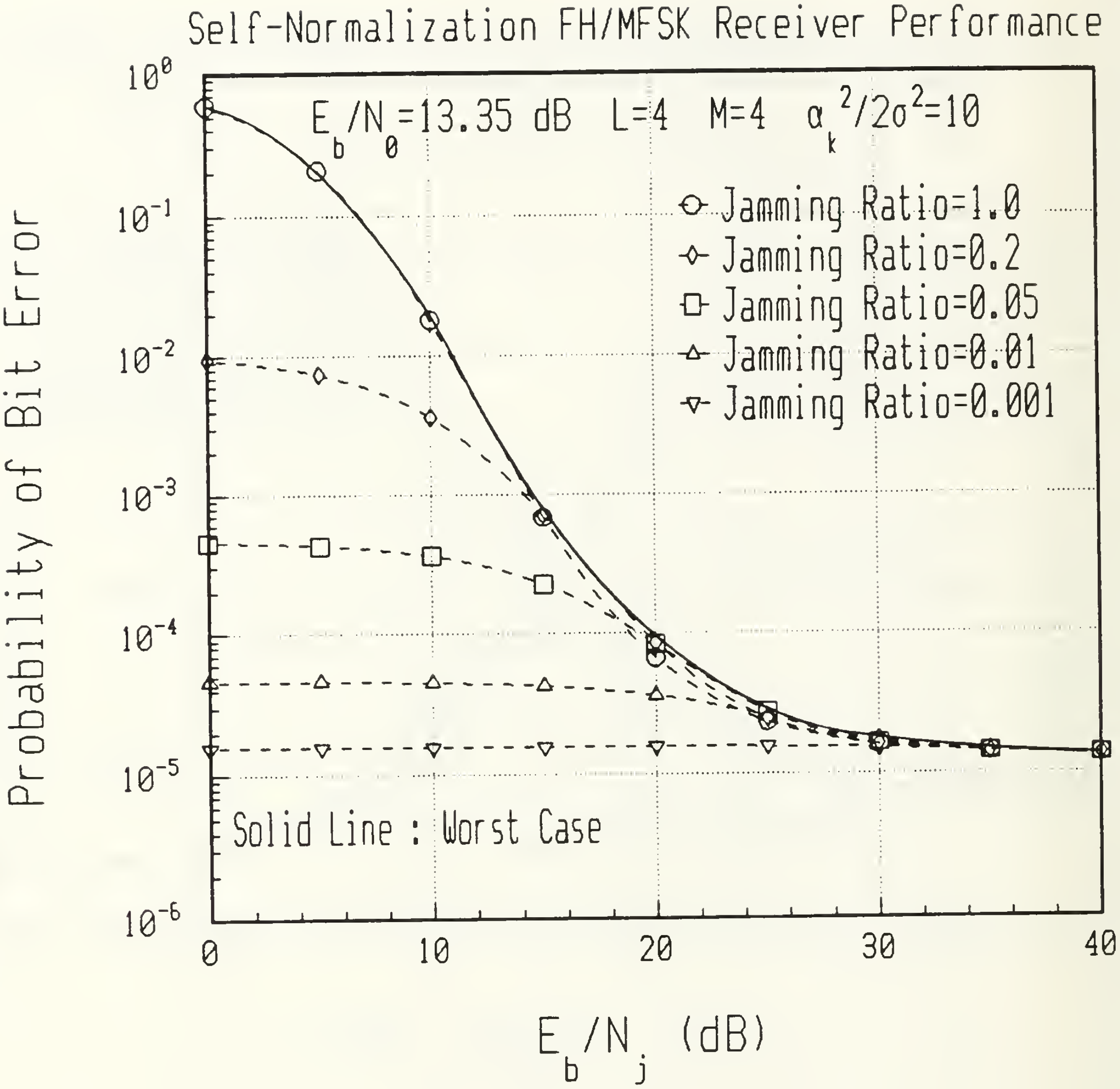


Figure 4.20: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

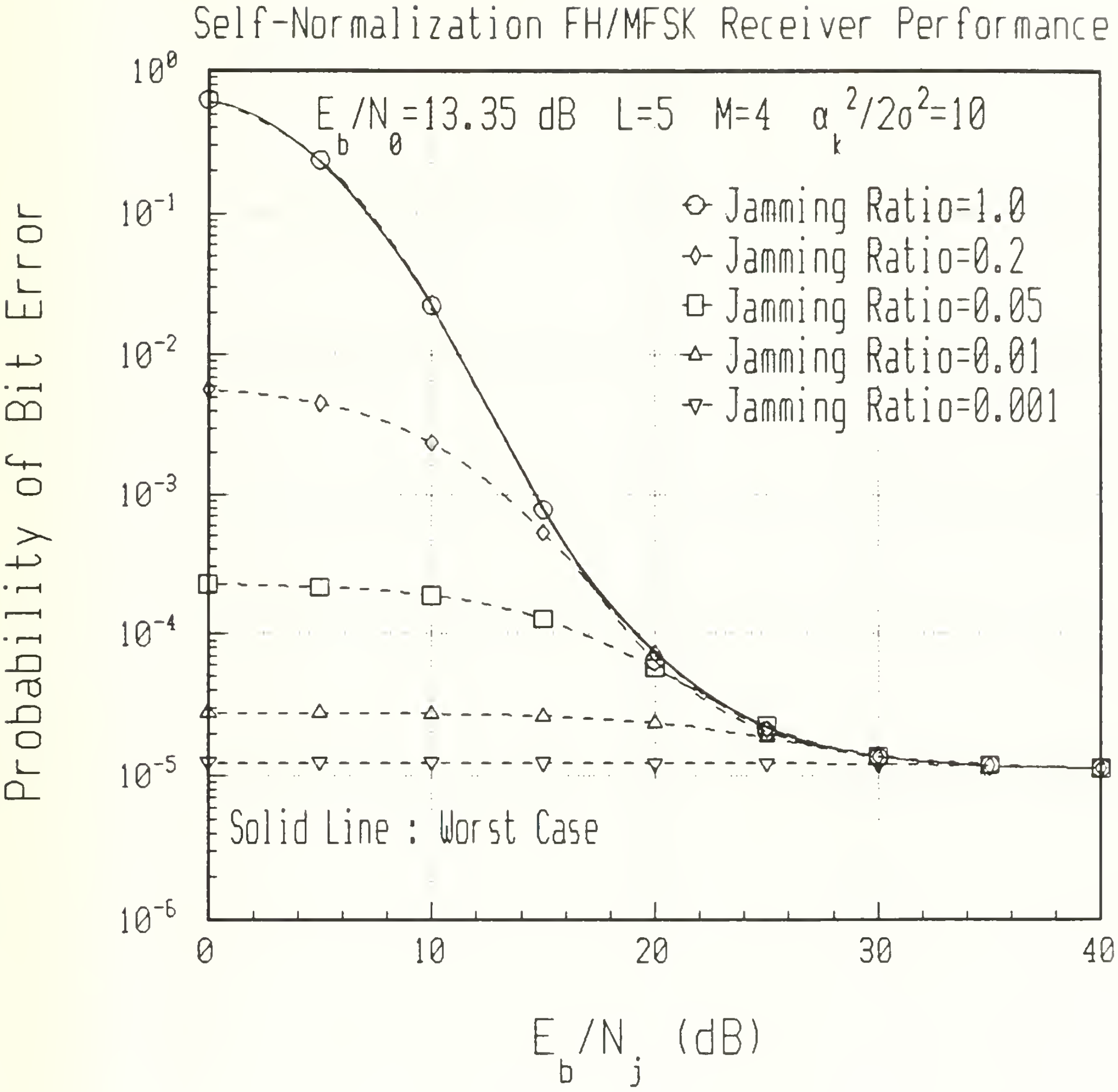


Figure 4.21: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 5, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

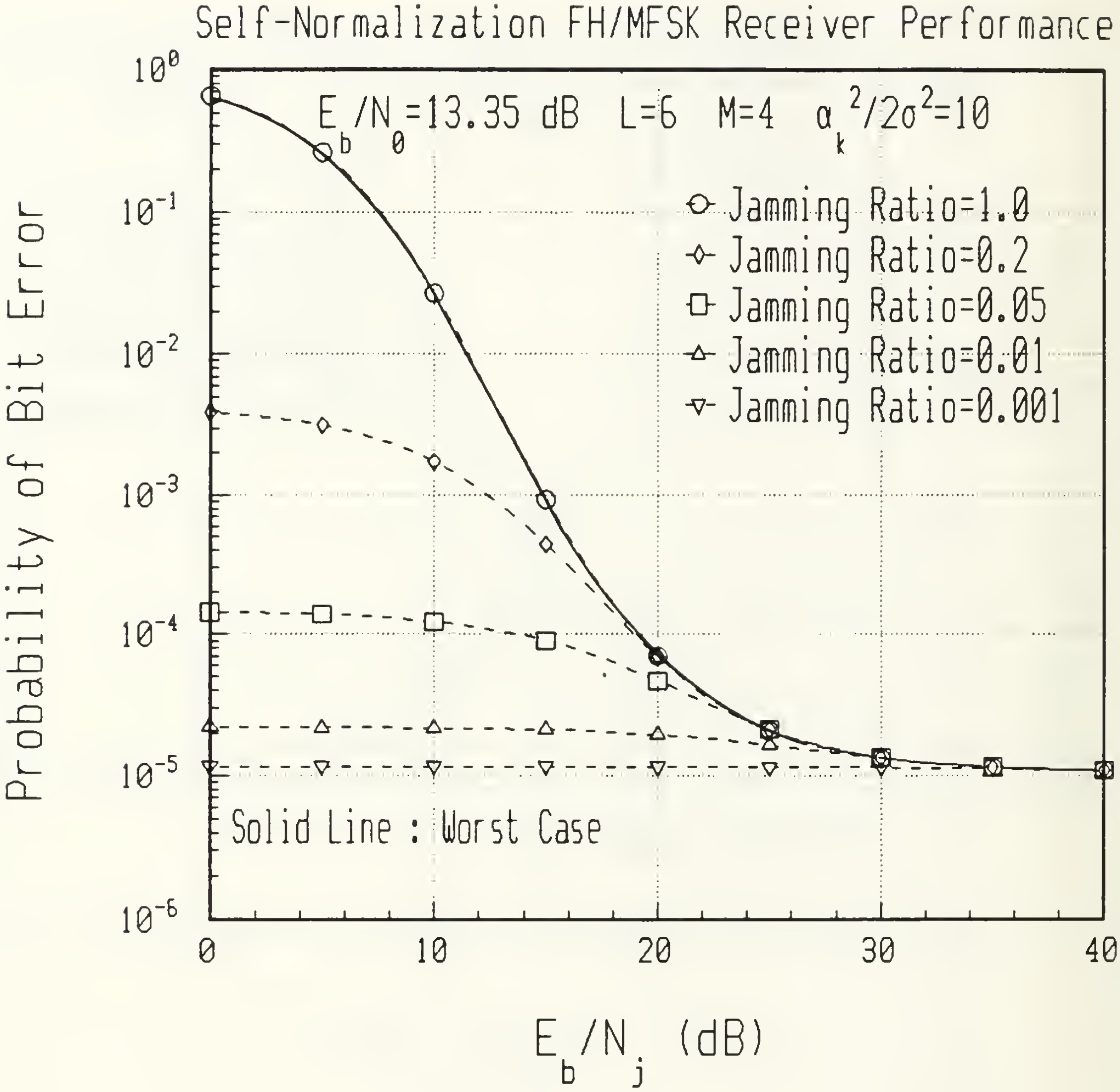


Figure 4.22: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 6, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 10

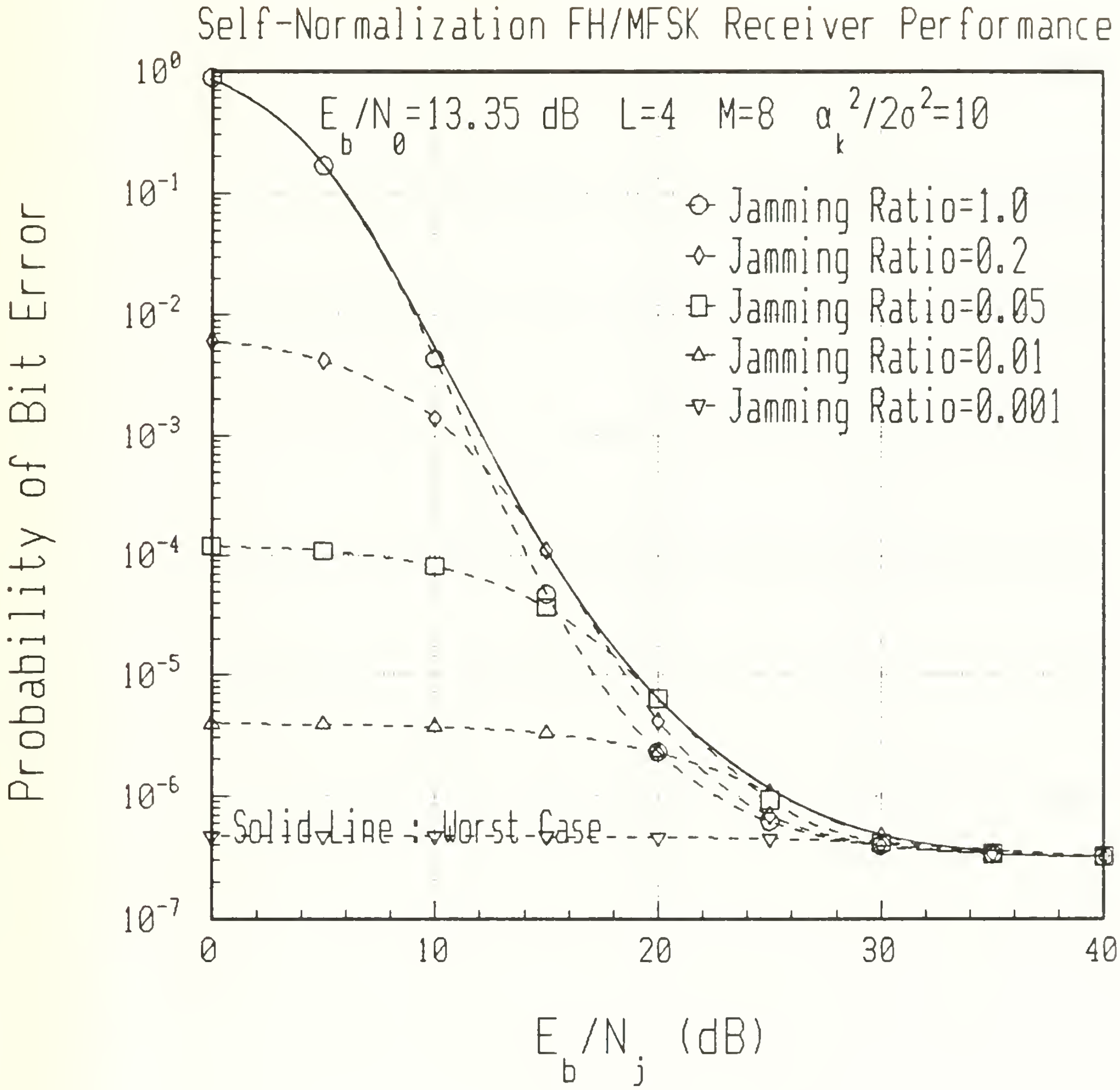


Figure 4.23: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 8 and Direct-to-Diffuse Signal Power Ratio = 10

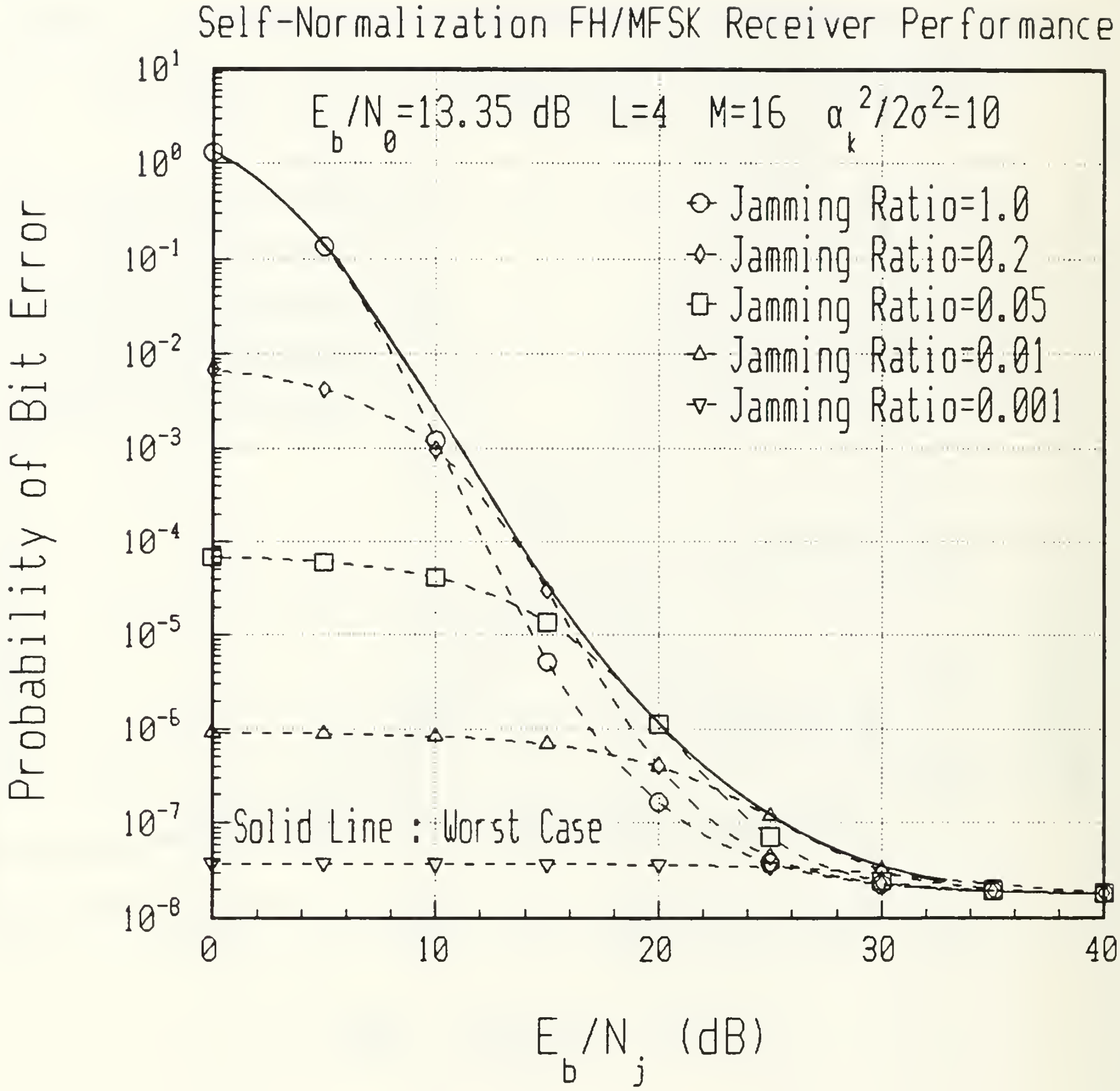


Figure 4.24: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 16 and Direct-to-Diffuse Signal Power Ratio = 10

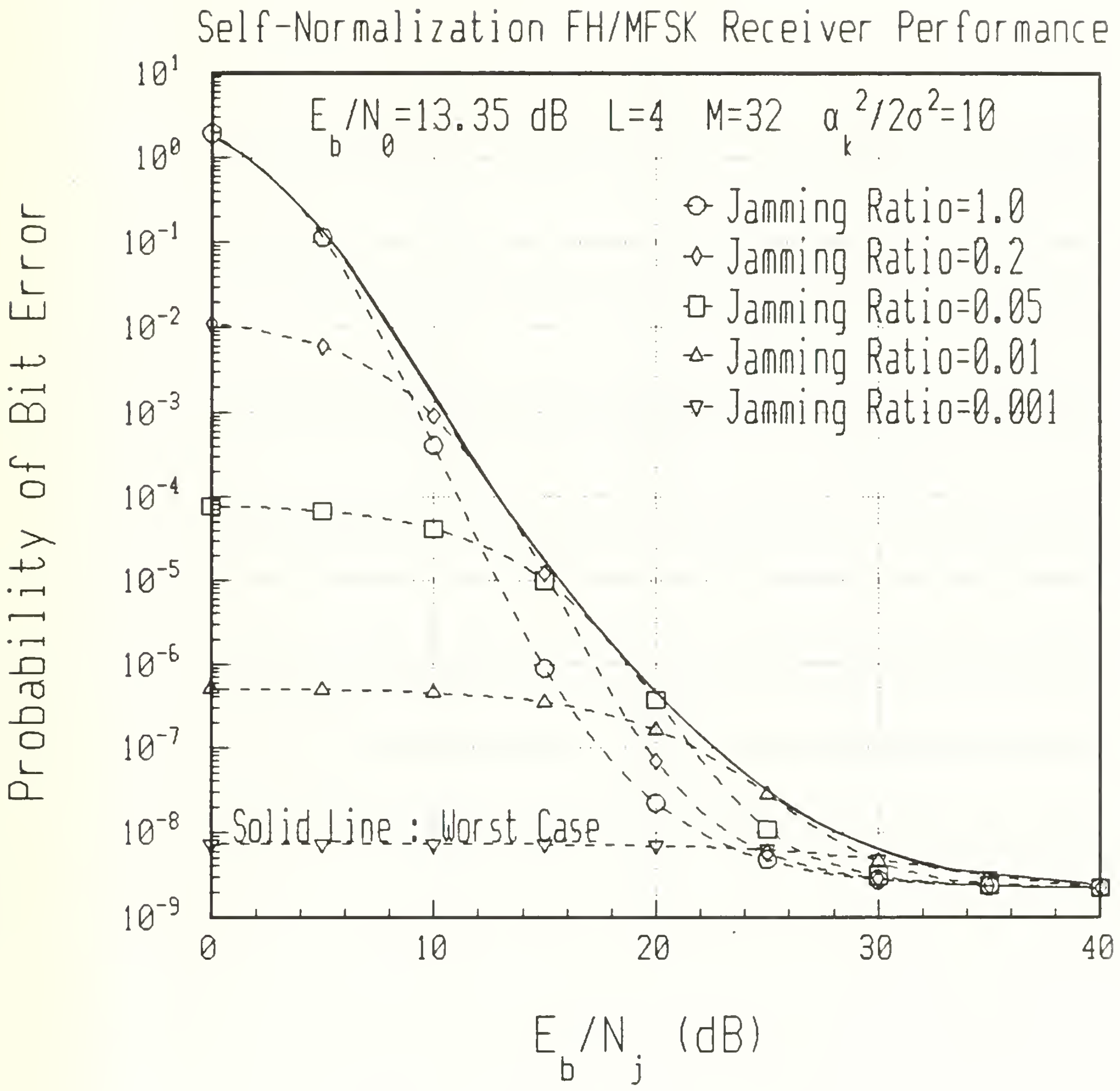


Figure 4.25: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 13.35 dB, Hopping Number per Symbol = 4, Order of Modulation = 32 and Direct-to-Diffuse Signal Power Ratio = 10

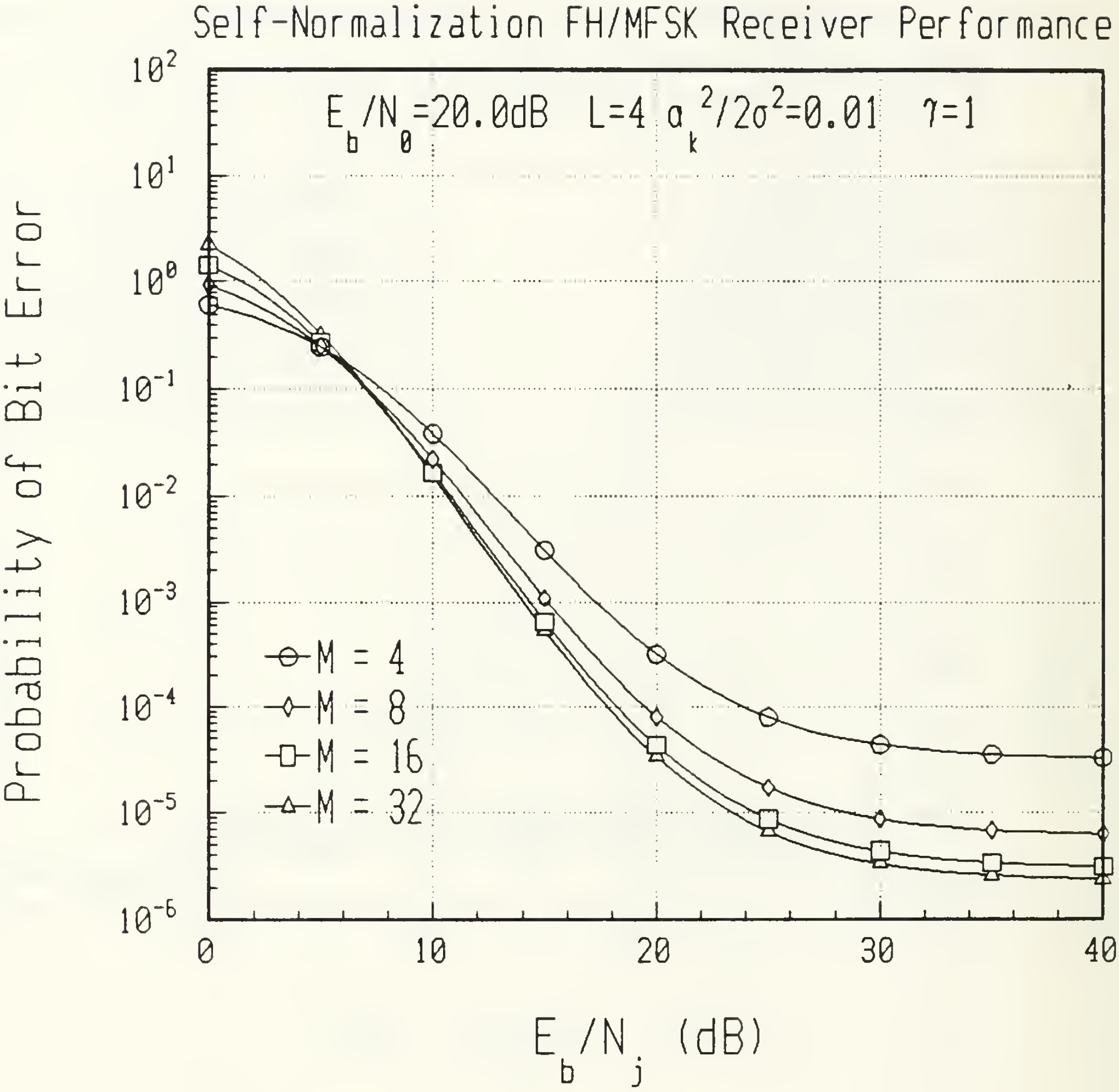


Figure 4.26: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 20.0dB, Hopping Number per Symbol = 4, Jamming Ratio = 1 and Direct-to-Diffuse Signal Power Ratio = 0.01

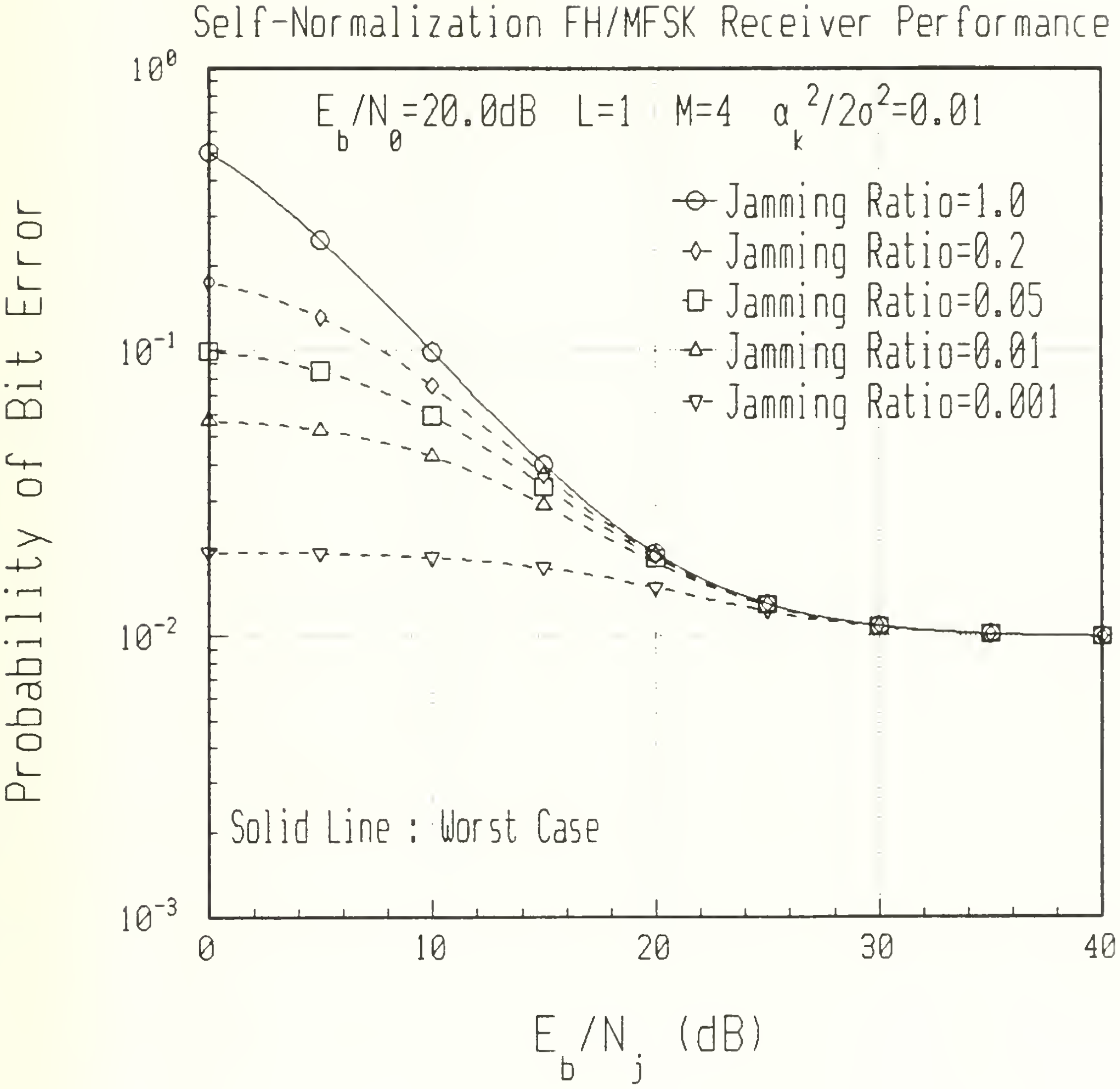


Figure 4.27: The Performance of the Self-Normalization FFH/MFSK non-coherent Receiver at Signal-to-Thermal Noise Ratio = 20.0 dB, Hopping Number = 1, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 0.01

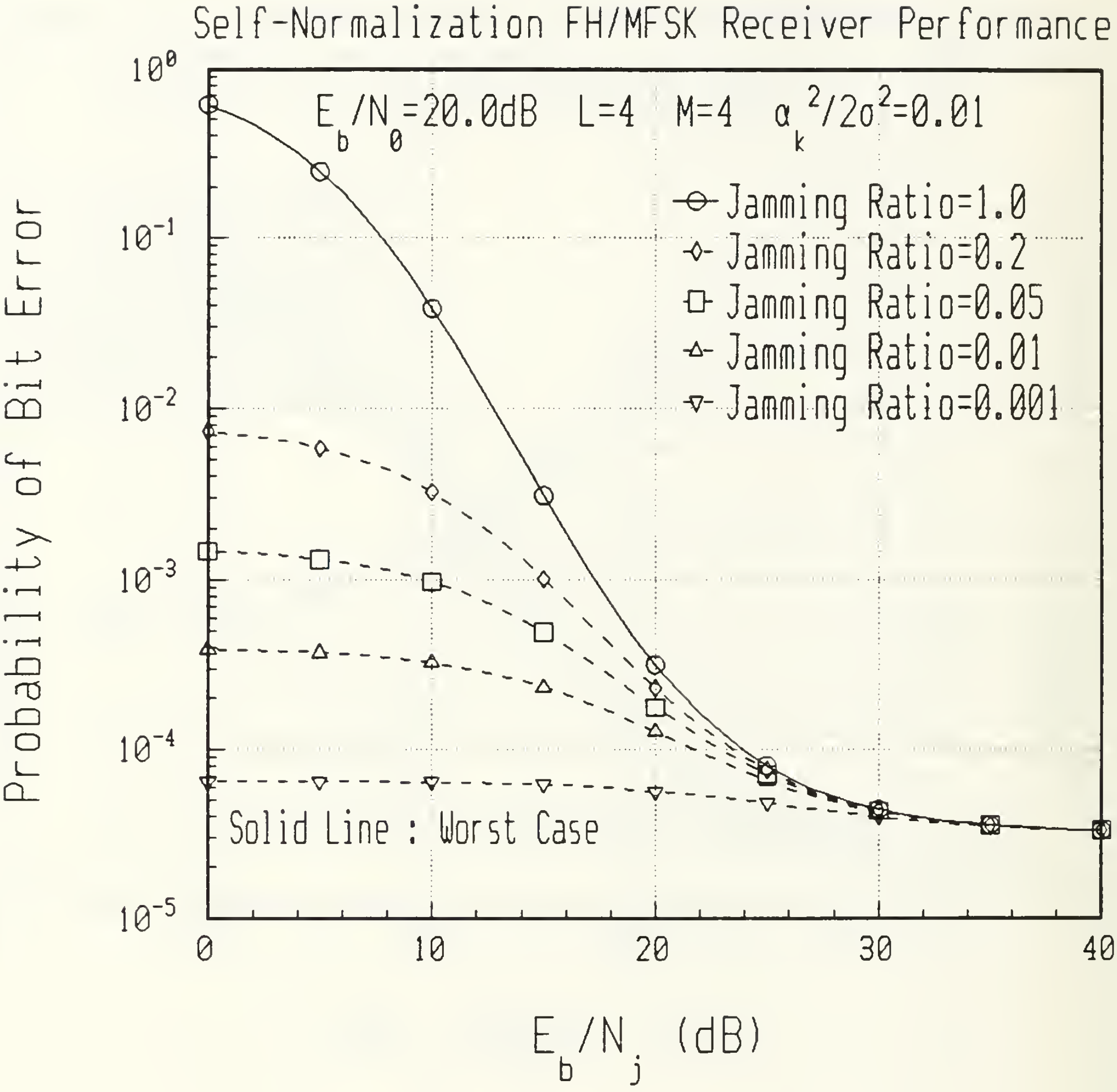


Figure 4.28: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 20.0dB, Hopping Number = 4, Order of Modulation = 4 and Direct-to-Diffuse Signal Power Ratio = 0.01

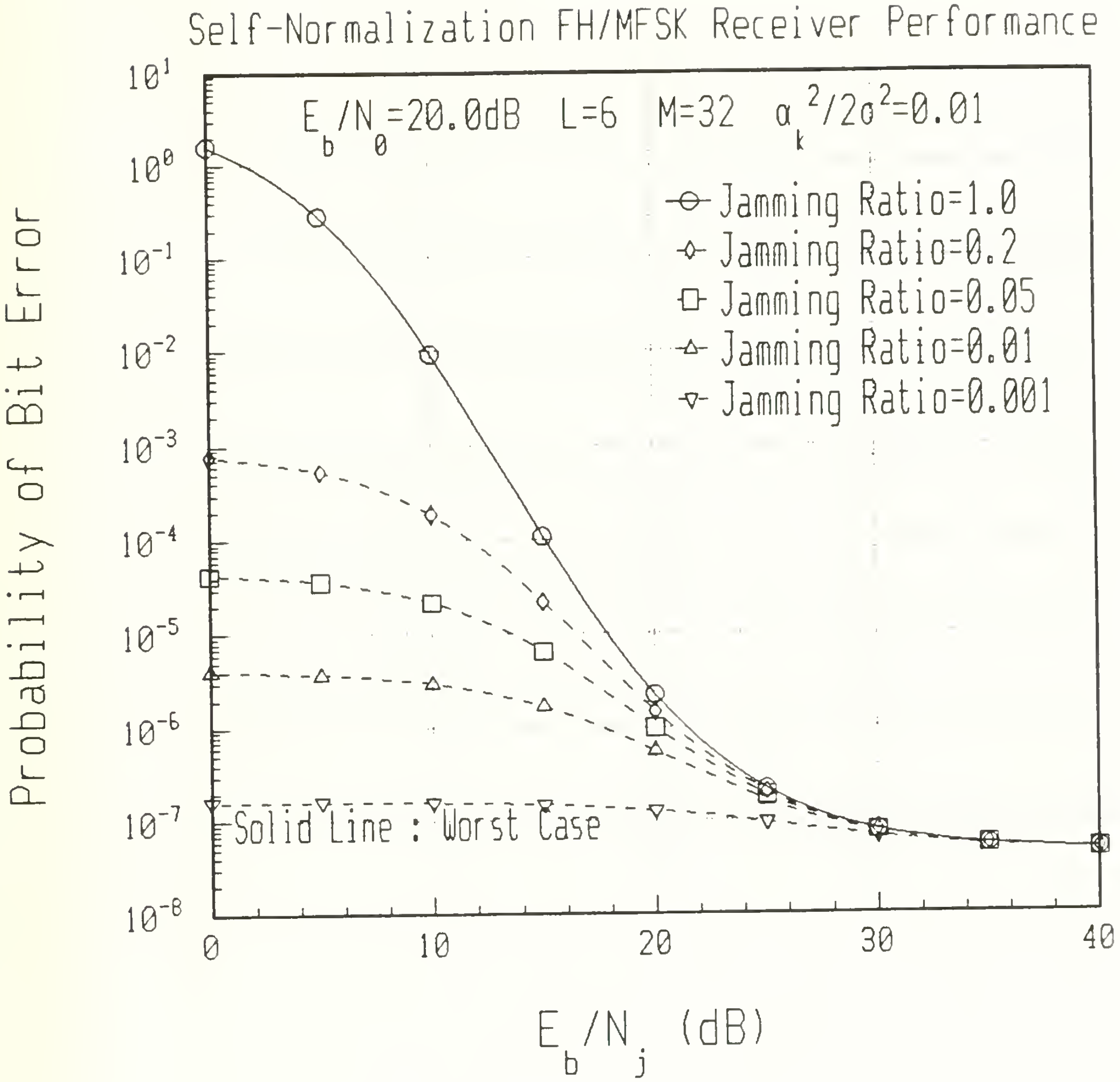


Figure 4.29: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 20.0dB, Hopping Number = 6, Order of Modulation = 32 and Direct-to-Diffuse Signal Power Ratio = 0.01

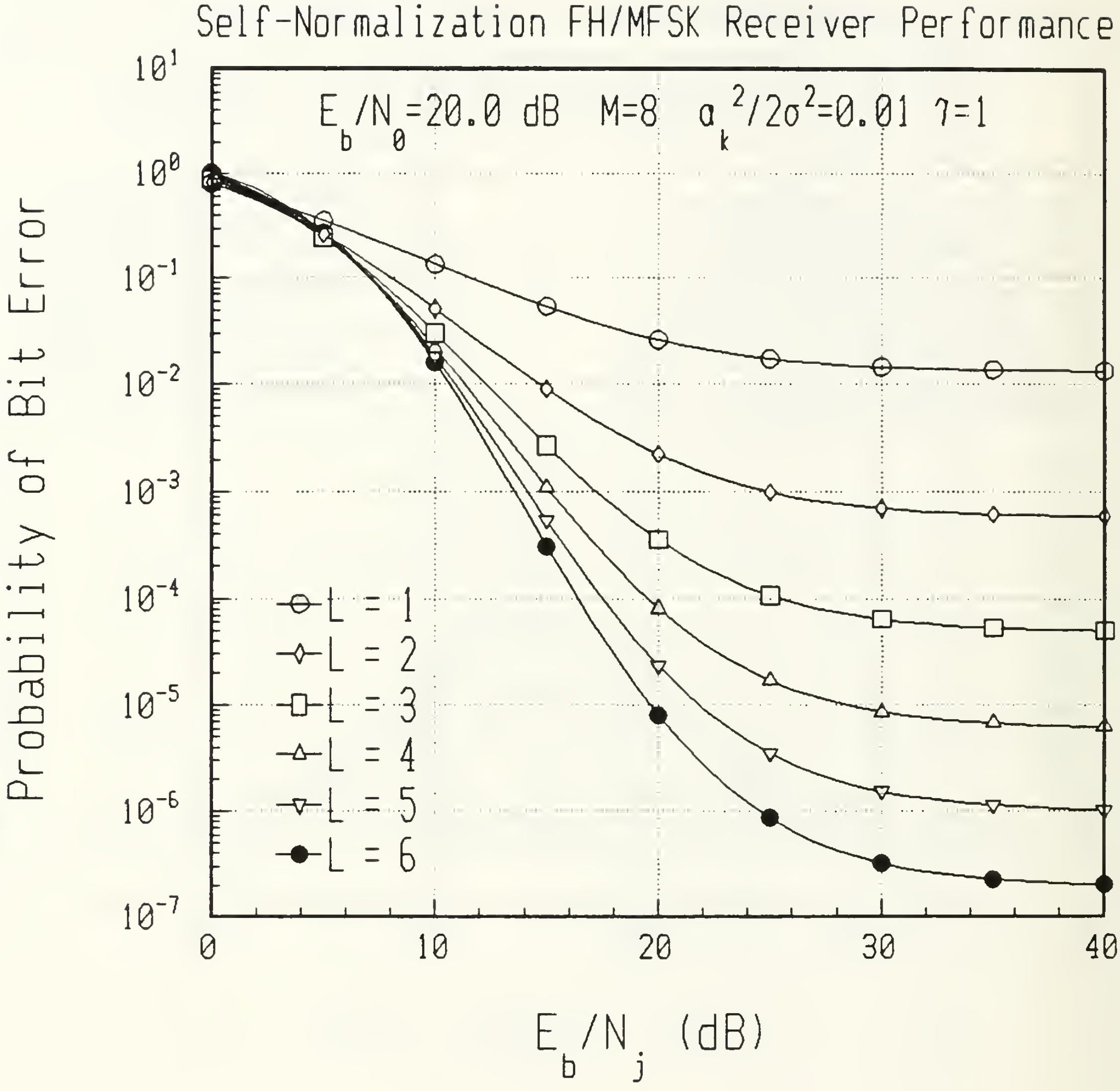


Figure 4.30: The Performance of the Self-Normalization FFH/MFSK noncoherent Receiver at Signal-to-Thermal Noise Ratio = 20.0dB, Jamming Ratio = 1, Order of Modulation = 8 and Direct-to-Diffuse Signal Power Ratio = 0.01

V. CONCLUSION

The probability of bit error performance for a conventional noncoherent FFH/MFSK receiver has been obtained for Rayleigh- and Rician-faded channel with partial-band jamming. The effect of fading is detrimental for both fading models, but fast frequency-hopping in general provides a means to overcome, at least partially, fading effects. However, the conventional receiver is severely affected by worst case partial-band jamming. It does not show any improvement in worst case performance with high diversity or high order of modulation. The probability of bit error of the conventional receiver in worst case partial-band jamming is always between 10^{-2} and 10^{-3} at E_b/N_j of 20 dB regardless of E_b/N_0, the order of modulation, diversity factor, or direct-to diffuse power ratio. The probability of bit error of the self-normalization receiver in worst case partial-band jamming is the same as that of conventional receiver for slow frequency-hopping (L=1). However, for fast frequency-hopping, the performance becomes better and better with increasing diversity factors: the probability of bit error is 5×10^{-4} at L=2, 2×10^{-4} at L=3, 10^{-4} at L=4 and 8×10^{-5} at L=5 and L=6. When the diversity factor is 6, the worst case performance is almost same as that of full band jamming case (jamming ratio 1.0). At this point the jammer does not need any information about the communicator to optimize the jamming effect by managing the jamming power.

The self-normalization receiver with diversity provides very good immunity to worst case partial-band jamming and has a performance similar to that of the noise-normalization receiver. The order of modulation also improves the overall performance of the self-normalization receiver, but partial-band jamming affects higher orders of modulation more than the lower ones. Thus, if the communicator uses a higher order of modulation and wants to suppress all partial-band jamming effects, he should use a higher hopping rate.

APPENDIX A

```
% This is the Matlab program to produce the probability of bit  %
error of
% Conventional FH/MFSK quadratic receiver.
% Lee,Kang Yeun  (Major ROKAF)

  clear
% 3 signal-to-noise ratios to be implemented (13.35 db, 16 db and
% 20 db)
  SNR=[21.62718524D0 39.81071706D0 100.0D0];
% 2 direct-to-diffuse signal power ratios (0.01 and 10)
  DDR=[0.01D0 10.0D0];
% 4 jamming ratio (1, 0.1, 0.01 and 0.001)
  JRATIO=[1.0D0 0.1D0 0.01D0 0.001D0];
  data=zeros(8,51);
% set the thermal plus jamming noise power 1
  rhok1=1;
% basic sequence length is 1024
  lim=1024;
  n=2*lim;
  zpad=zeros(1,lim);
  opad=ones(1,lim);
% hopping number (1 to 6)
  for  L=1:6,
  for j1=1:2,  psi=DDR(j1);
  for j2=1:3,  nu=SNR(j2);
  for j3=1:4,  gama=JRATIO(j3);
% the order of modulation (M-ary : 4, 8, 16, 32-ary)
  for j4=2:5;  M=2^j4;
  for j5=0:2:50, iota=10.0D0^(0.1D0*j5);
% diffuse signal power ( rho )
  rho=nu*gama*iota/(nu+gama*iota)/(psi+1)*j4/L;
% direct signal power ( alpha )
  alpha=rho*psi;
% thermal noise power ( rhok2 )
  rhok2=iota*gama/(iota*gama+nu);
% set the sequence limit roughly based on the singal-to-noise ratio
% (nu)
  limx=80*nu ;
% divide the range by 100 to check the proper limit
  delta=limx/100;
% select almost the middle value of the increment to compute the
% sequence
% of the probability density function.
```

```
  x=0.49*delta:delta:limx-0.51*delta;
  g=exp(-(x+2*L*alpha)/2/(rhok1+rho))/2/(rhok1+rho). ...
     * x/2/L/alpha).^...
    ((L-1)/2).*abs(j^(L-1)*bessel(L-1,j*sqrt(2*L*alpha*x) ...
    /(rhok1+rho)))*delta;
% check the sequence whether it has invalid sequence or not
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
% if the sequence has only 0s, the limit is too wide. Thus set the
% first
% increment as the new limit
  if    all(g==0)==1,
  gmin=1;
% if the sequence is valid, check the point around 1.0e-6 to select
% the
% new limit
  else, [xx,gmax]=max(g);
  g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end
  limx=gmin*delta;
% if the new limit is first or second value of the increment
% redo above procedure
  while gmin <= 2 ,
  delta=limx/100; x=0.5*delta:delta:limx-0.5*delta;
  g=exp(-(x+2*L*alpha)/2/(rhok1+rho))/2/ ...
    (rhok1+rho).*(x/2/L/alpha).^ ...
    ((L-1)/2).*abs(j^(L-1)*bessel(L-1,j*sqrt(2*L*alpha*x) ...
     /(rhok1+rho)))*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if    all(g==0)==1,
  gmin=1;
  else, [xx,gmax]=max(g);
  g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end
  limx=gmin*delta;
  end
% after setting the limit, whole range is divided by 1024
% to produce the sequence
  delta=limx/lim;x=0.49*delta:delta:limx-0.51*delta;
% if jamming ratio is 1, the computation is simpler than others
  if j3 == 1,
  gs=-(x+2*L*alpha)/2/(rhok1+rho)-log(2*(rhok1+rho))
  +log(x/2/L/alpha)* ...
  (L-1)/2+log(abs(j^(L-1)*bessel(L-1,j*sqrt(2*L*alpha*x) ...
  /(rhok1+rho))));
  gs=exp(gs)*delta;
  fi=find(isnan(gs));
  gs(fi)=zeros(1,length(fi));
  if all(gs==0)==1,
  gs=zeros(1,lim);
  gs(1)=1;
  else,gs=gs/sum(gs);
  end
  ps=log(1/2/rhok1)*L+log(x)*(L-1)-log(fact(L-1)) - x/2/rhok1;
  ps=exp(ps)*delta;
  fi=find(isnan(ps));
  ps(fi)=zeros(1,length(fi));
  if all(ps==0)==1,
  ps=ones(1,lim);
  else,ps=ps/sum(ps);
  ps=cumsum(ps);
  ps=ps/max(ps);
  end
  data(count,j5+1)=M/2/(M-1)*(1-sum(gs.*ps.^(M-1)));
  else
% if jamming ratio is not 1 , the hopping number has to be
%  considered
  for i = 0 : L
% if no hop is jammed
  if i == 0,
  delta=limx/100;
  x=0.5*delta:delta:limx-0.5*delta;
  g=-(x+2*L*alpha)/2/(rhok2+rho)-log(2*(rhok2+rho)) ...
  +log(x/2/L/alpha)* ...
  (L-1)/2+log(abs(j^(L-1)*bessel(L-1,j*sqrt(2*L*alpha*x) ...
  /(rhok2+rho))));
  g=exp(g)*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if all(g==0)==1,
  gmin=1;
  else,[xx,gmax]=max(g);g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end
  vlimx=gmin*delta;
% check the sequence limit again
  while gmin <= 2,
  delta=vlimx/100;
  x=0.5*delta:delta:vlimx-0.5*delta;
  g=-(x+2*L*alpha)/2/(rhok2+rho)-log(2*(rhok2+rho)) ...
  +log(x/2/L/alpha)* ...
  (L-1)/2+log(abs(j^(L-1)*bessel(L-1,j*sqrt(2*L*alpha*x) ...
  /(rhok2+rho))));
  g=exp(g)*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if all(g==0)==1,
  gmin=1;
  else,
  [xx,gmax]=max(g);g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end
  vlimx=gmin*delta;
  end
  delta=vlimx/lim;
  x=0.49*delta:delta:vlimx-0.51*delta;
  gs=-(x+2*L*alpha)/2/(rhok2+rho)-log(2*(rhok2+rho)) ...
    +log(x/2/L/alpha)* ...
  (L-1)/2+log(abs(j^(L-1)*bessel(L-1,j*sqrt(2*L*alpha*x) ...
  /(rhok2+rho))));
  gs=exp(gs)*delta;
  fi=find(isnan(gs));
  gs(fi)=zeros(1,length(fi));
  if all(gs==0)==1,
  gs=zeros(1,lim);
  gs(1)=1;
  else,
  gs=gs/sum(gs);
  end
  ps=-log(2*rhok2)*L+log(x)*(L-1)-log(fact(L-1)) - x/2/rhok2;
  ps=exp(ps)*delta;
  fi=find(isnan(ps));
  ps(fi)=zeros(1,length(fi));
  if all(ps==0)==1,
  ps=ones(1,lim);
  else, ps=ps/sum(ps);
  ps=ps/sum(ps);
  ps=cumsum(ps);
  ps=ps/max(ps);
  end
% if all hops are jammed
  elseif i == L,
  delta=limx/100;
  x=0.5*delta:delta:limx-0.5*delta;
  g=-(x+2*i*alpha)/2/(rhok1+rho)-log(2*(rhok1+rho)) ...
  +log(x/2/i/alpha)* ...
  (i-1)/2+log(abs(j^(i-1)*bessel(i-1,j*sqrt(2*i*alpha*x) ...
  /(rhok1+rho))));
  g=exp(g)*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if all(g==0)==1,
  gmin=1;
  else,
  [xx,gmax]=max(g);g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end
  ulimx=gmin*delta;
% check the sequence limit again
  while gmin <=2,
  delta=ulimx/100;
  x=0.5*delta:delta:ulimx-0.5*delta;
  g=-(x+2*i*alpha)/2/(rhok1+rho)-log(2*(rhok1+rho)) ...
  +log(x/2/i/alpha)* ...
  (i-1)/2+log(abs(j^(i-1)*bessel(i-1,j*sqrt(2*i*alpha*x) ...
   /(rhok1+rho))));
  g=exp(g)*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if all(g==0)==1,
  gmin=1;
  else,
  [xx,gmax]=max(g);
  g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end

  ulimx=gmin*delta;

  end
  delta=ulimx/lim;
  x=0.49*delta:delta:ulimx-0.51*delta;
  gs=-(x+2*i*alpha)/2/(rhok1+rho)-log(2*(rhok1+rho)) ...
   +log(x/2/i/alpha)* ...
  (i-1)/2+log(abs(j^(i-1)*bessel(i-1,j*sqrt(2*i*alpha*x) ...

  /(rhok1+rho))));
  gs=exp(gs)*delta;
  fi=find(isnan(gs));
  gs(fi)=zeros(1,length(fi));
  if all(gs==0)==1,
  gs=zeros(1,lim);
  gs(1)=1;
  else,
  gs=gs/sum(gs);
  end
  ps=-log(2*rhok1)*i+log(x)*(i-1)-log(fact(i-1)) - x/2/rhok1;
  ps=exp(ps)*delta;
  if    all(ps==0) ==1,
  ps= ones(1,lim);
  else,
  fi= find(isnan(ps));
  ps(fi)=zeros(1,length(fi));
  ps = cumsum(ps);
  ps = ps/max(ps);
  end
  else
% if i hops are jammed and L-i hops are not jammed
% check the sequence limit again
  rhok=rhok1;
  delta=limx/100;
  x=0.5*delta:delta:limx-0.5*delta;
  g=-(x+2*i*alpha)/2/(rhok+rho)-log(2*(rhok+rho)) ...
  +log(x/2/i/alpha)* ...
  (i-1)/2+log(abs(j^(i-1)*bessel(i-1,j*sqrt(2*i*alpha*x) ...
  /(rhok+rho))));
  g=exp(g)*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if all(g==0)==1,
  gmin=1;
  else,
  [xx,gmax]=max(g);
  g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end


  wlimx=gmin*delta;
  while gmin <=2,
  delta=wlimx/100;
  x=0.5*delta:delta:wlimx-0.5*delta;
  g=-(x+2*i*alpha)/2/(rhok+rho)-log(2*(rhok+rho)) ...
  +log(x/2/i/alpha)* ...
  (i-1)/2+log(abs(j^(i-1)*bessel(i-1,j*sqrt(2*i*alpha*x) ...
  /(rhok+rho))));
  g=exp(g)*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if all(g==0)==1,
  gmin=1;
  else,
  [xx,gmax]=max(g);
  g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end
  wlimx=gmin*delta;
  end
  rhok=rhok2;
  delta=limx/100;
  x=0.5*delta:delta:limx-0.5*delta;
  g=-(x+2*(L-i)*alpha)/2/(rhok+rho)-log(2*(rhok+rho))
   +log(x/2/(L-i)/alpha)* ...
  (L-i-1)/2+log(abs(j^(L-i-1)*bessel(L-i-1,j*sqrt(2*(L-i) ...
   *alpha*x)/ ...
  (rhok+rho))));
  g=exp(g)*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if all(g==0)==1,
  gmin=1;
  else,
  [xx,gmax]=max(g);
  g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end
  zlimx=gmin*delta;

% check the sequence limit again
  while gmin <=2,
  delta=zlimx/100;
  x=0.5*delta:delta:zlimx-0.5*delta;
  g=-(x+2*(L-i)*alpha)/2/(rhok+rho)-log(2*(rhok+rho)) ...
  +log(x/2/(L-i)/alpha)* ...
  (L-i-1)/2+log(abs(j^(L-i-1)*bessel(L-i-1,j*sqrt(2*(L-i) ...
  *alpha*x)/(rhok+rho))));
  g=exp(g)*delta;
  fi=find(isnan(g));
  g(fi)=zeros(1,length(fi));
  if all(g==0)==1,
  gmin=1;
  else,
  [xx,gmax]=max(g);
  g(1:gmax)=ones(1,gmax)*xx;
  [xx,gmin]=min((g-1.0e-6).^2);
  end
  zlimx=gmin*delta;
  end
% produce the sequency of jammed hop of signal branch
  delta=wlimx/lim;
  x=0.49*delta:delta:wlimx-0.51*delta;
  g1=-(x+2*i*alpha)/2/(rhok1+rho)-log(2*(rhok1+rho)) ...
  +log(x/2/i/alpha)* ...
  (i-1)/2+log(abs(j^(i-1)*bessel(i-1,j*sqrt(2*i*alpha*x) ...
   /(rhok1+rho))));
  g1=exp(g1)*delta;
  fi=find(isnan(g1));
  g1(fi)=zeros(1,length(fi));
  if all(g1==0)==1,
  g1=zeros(1,lim);
  g1(1)=1;
  else,
  g1=g1/sum(g1);
  end
% produce the sequence of nonjammed hop of signal branch
  if wlimx >= 100*zlimx
  gs=[g1 zpad];
  else
  g2=-(x+2*(L-i)*alpha)/2/(rhok2+rho)-log(2*(rhok2+rho)) ...
  +log(x/2/(L-i) ...
  /alpha)*(L-i-1)/2+log(abs(j^(L-i-1)*bessel(L-i-1,  ...
  j*sqrt(2*(L-i) ...
  *alpha*x)/(rhok2+rho))));
  g2=exp(g2)*delta;
  fi=find(isnan(g2));
  g2(fi)=zeros(1,length(fi));
  if all(g2==0)==1,
  g2=zeros(1,lim);
  g2(1)=1;
  else,
  g2=g2/sum(g2);
  end

% produce the sequence of signal branch
  gs = abs(ifft(fft(g1,n).*fft(g2,n),n));
  gs = gs/sum(gs);
  end

% produce the sequence of jammed hop of noise branch
  p1=-log(2*rhok1)*i+log(x)*(i-1)-log(fact(i-1))-x/2/rhok1;
  p1=exp(p1)*delta;
  fi=find(isnan(p1));
  p1(fi)=zeros(1,length(fi));
  if all(p1==0)==1,
  p1=zeros(1,lim);
  p1(1)=1;
  else,
  p1=p1/sum(p1);
  end
% produce the sequence of nonjammed hop of noise branch
  p2=-log(2*rhok2)*(L-i)+log(x)*(L-i-1)-log(fact(L-i-1))-x/2/rhok2;
  p2=exp(p2)*delta;
  fi=find(isnan(p2));
  p2(fi)=zeros(1,length(fi));
  if all(p2==0)==1,
  p2=zeros(1,lim);
  p2(1)=1;
  else,
  p2=p2/sum(p2);
  end
% produce the sequence of noise branch
  ps = abs(ifft(fft(p1,n).*fft(p2,n),n));
  ps = cumsum(ps);
  ps = ps/max(ps);
  end
```

```
% compute the probability of bit error using sequence
  tdata(j5+1)=tdata(j5+1)+bino(L,i)*gama^i*(1-gama)^(L-i) ...
  *M/2*(1-sum(gs.*ps.^(M-1)));
  end, end, end

  data=[data;td];

  save datafile data;

  end,end,end,end,end
```

APPENDIX B

```
% This is the program to produce the probability of bit error of
% Self-Normalization FH/MFSK Receiver
% Lee, Kang Yeun (Major, ROKAF)

% three special routines
% fact : factorial routine
% bino : binomial coefficient routine
% ereval : error probability computation routine using fft

% 3 signal-to-noise ratio(13.35db, 16db, 20db)
  SNR=[21.62718524D0 39.81071706D0 100.0D0];
% 3 direct-to-diffuse signal power ratio (0.01, 1, 10)
  DDR=[0.01D0 1.0D0 10.0D0];
% 4 jamming ratio
  JRATIO1=[1.0D0 0.2D0 0.1D0 0.05D0 0.01D0];
  JRATIO2=[1.0D0 0.2D0 0.05D0 0.01D0 0.001D0];

% Bandwidth-to-Symbol Rate Ratio (BW/Rs : 1  2 )
  mu=1.0D0;

% M-ary
  M=16.0D0;
% Multichannel factor  w=logM/log2
  omega=log(M)/log(2.0D0);

% Bit SNR  13.35 db
  nu=SNR(1);

  count=1;

% Hopping number (1 to 6)
  for J1=1:6
  L=J1;

% Direct-to-Diffuse signal ratio ( 0.01  1  10 )
  for J3=1:3
  psi=DDR(J3);

% Direct SNR of NOJAMMING CASE
  rho=omega*nu*psi/mu/(psi+1.0D0)/L;
% Diffuse SNR of NOJAMMING CASE
  xi=omega*nu/mu/(psi+1.0D0)/L;

% produce the sequence of nojamming case
```

```
z=-0.999D0:0.002D0:0.999D0;
NOJAM=exp(rho*(z-1.0D0)./(2.0D0+xi*(1.0D0-z))). ...
*(1.0D0+z).^(M-2.0D0)...
./(2.0D0+xi*(1.0D0-z)).*((2.0D0*(1.0D0+xi))./ ...
(xi*(1.0D0-z)+2.0D0)...
.*(1.0D0+rho*(1.0D0+z)./((2.0D0+xi*(1.0D0-z)) ...
*(1.0D0+xi)))+M-2.0D0);


z=0.001D0:0.002D0:0.999D0;
B=rho*z./((1.0D0+xi*(1.0D0-z))*(1.0D0+xi));
E=exp(rho*(z-1.0D0)./(1.0D0+xi*(1.0D0-z)));

TT=zeros(M-1,500);
for N=0:M-2
TT(N+1,:)=fact(M-2.0D0)/fact(M-2.0D0-N)/fact(N)^2*B.^N;
end
T1=sum(TT).*((1.0D0-z)*(1.0D0+xi)).^(M-2).*z. ...
*(M-2.0D0)./(1.0D0+z)...
./(1.0D0+xi*(1.0D0-z)).^(M-1).*E;

TT=zeros(M-2,500);
for N=0:M-3
TT1=zeros(M-1-N,500);
for IQ=0:M-2-N
TT1(IQ+1,:)=fact(M-2.0D0-N)/fact(M-2.0D0-N-IQ)/fact(IQ)^2*B.^IQ;
end
TEMP=2.0D0*((1.0D0-z)*(1.0D0+xi)).^(M-3-N).*(1.0D0+xi).*z.^2 ...
    .*(1.0D0+z).^(N-1).*(M-2.0D0-N);
TEMP=TEMP./(1.0D0+xi*(1.0D0-z)).^(M-1.0D0-N).*E.*sum(TT1);
TT1=zeros(M-2-N,500);
for IQ=0:M-3-N
TT1(IQ+1,:)=fact(M-3.0D0-N)/fact(M-3.0D0-N-IQ)/fact(IQ)^2*B.^IQ;
end
TEMP1=((1.0D0-z)*(1.0D0+xi)).^(M-3-N).*z.*(1.0D0+z) ...
   .^N.*(M-2.0D0)./(1.0D0+xi*(1.0D0-z)).^(M-2-N).*E;
if IQ == 0,
TT(N+1,:)=TEMP+TEMP1.*TT1;
else
TT(N+1,:)=TEMP+TEMP1.*sum(TT1);
end
end
T1=T1+sum(TT);

% sequence of nojamming case
NOJAM(501:1000)=NOJAM(501:1000)-T1;

  Ps=zeros(5,41);

% jamming ratio
  for J4=1:5
  if J3==3
  gamma=JRATIO2(J4);
  else
  gamma=JRATIO1(J4);
  end

% Signal-to-Jamming noise ratio ( 0 db - 40 db )

  for J5=0:40
  iota=10.0D0^(0.1D0*J5);

% Direct SNR of JAMMING case
  rho=omega*nu*iota*psi*gamma/mu/(psi+1.0D0)/(iota*gamma+nu)/L;
% Diffuse SNR of JAMMING case
  xi=omega*nu*iota*gamma/mu/(psi+1.0D0)/(iota*gamma+nu)/L;

% produce the sequence when the signal is jammed by noise jamming

z=-0.999D0:0.002D0:0.999D0;
JAM=exp(rho*(z-1.0D0)./(2.0D0+xi*(1.0D0-z))). ...
*(1.0D0+z).^(M-2.0D0)...
 ./(2.0D0+xi*(1.0D0-z)). ...
*( (2.0D0*(1.0D0+xi))./(xi*(1.0D0-z)+2.0D0)...
.*(1.0D0+rho*(1.0D0+z)./((2.0D0+xi*(1.0D0-z)) ...
*(1.0D0+xi)))+M-2.0D0);


z=0.001D0:0.002D0:0.999D0;
B=rho*z./((1.0D0+xi*(1.0D0-z))*(1.0D0+xi));
E=exp(rho*(z-1.0D0)./(1.0D0+xi*(1.0D0-z)));

TT=zeros(M-1,500);
for N=0:M-2
TT(N+1,:)=fact(M-2.0D0)/fact(M-2.0D0-N)/fact(N)^2*B.^N;
end
T1=sum(TT).*((1.0D0-z)*(1.0D0+xi)).^(M-2). ...
*z.*(M-2.0D0)./(1.0D0+z)...
./(1.0D0+xi*(1.0D0-z)).^(M-1).*E;

TT=zeros(M-2,500);
for N=0:M-3
TT1=zeros(M-1-N,500);
for IQ=0:M-2-N
TT1(IQ+1,:)=fact(M-2.0D0-N)/fact(M-2.0D0-N-IQ)/fact(IQ)^2*B.^IQ;
end
TEMP=2.0D0*((1.0D0-z)*(1.0D0+xi)).^(M-3-N).*(1.0D0+xi).*z.^2 ...
    .*(1.0D0+z).^(N-1).*(M-2.0D0-N);
TEMP=TEMP./(1.0D0+xi*(1.0D0-z)).^(M-1.0D0-N).*E.*sum(TT1);
TT1=zeros(M-2-N,500);
for IQ=0:M-3-N
TT1(IQ+1,:)=fact(M-3.0D0-N)/fact(M-3.0D0-N-IQ)/fact(IQ)^2*B.^IQ;
```

```
end
TEMP1=((1.0D0-z)*(1.0D0+xi)).^(M-3-N).*z.*(1.0D0+z) ...
   .^N.*(M-2.0D0)./(1.0D0+xi*(1.0D0-z)).^(M-2-N).*E;
if IQ == 0,
TT(N+1,:)=TEMP+TEMP1.*TT1;
else
TT(N+1,:)=TEMP+TEMP1.*sum(TT1);
end
end
T1=T1+sum(TT);

% the sequence of jamming case
JAM(501:1000)=JAM(501:1000)-T1;


% 'bino' and 'ereval' are the functions for binomial and error
%  evaluation.

%  Ps(i) <= (M-1) * Pr( z < 0 )
%  Pb = (M/2)/(M-1)*Ps
%  Pb <= (M/2) * Pr( z < 0 )

% probability of symbol error
for i=0:L
Ps(J4,J5+1)=Ps(J4,J5+1)+bino(L,i)*gamma^i*(1-gamma)^(L-i) ...
            *ereval(NOJAM,L-i,JAM,i);
end

% probability of bit error
Ps(J4,J5+1)=(M/2.0D0)*Ps(J4,J5+1);

end
end

% Save the Results and Plot

save data Ps

end
end
```

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